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METHOD FOR SOLVING THE MULTI-CRITERIA NON-MARKOV PROBLEM OF PROJECT PORTFOLIO OPTIMIZATION

*The **subject** of the study in this paper is models and methods of optimization of the organization's project portfolio for the planning period, considering the effects of the previously made decisions. Project portfolio optimization is one of the responsible and complex tasks by company's top management solves. Based on the analysis of the known works in the field, the research **purpose** is described: to create a method that would allow solve the problem of multi-criteria project portfolio optimization for the planned period, considering the aftereffects of the previously made decisions. The research **tasks** are to enhance the method for solving the project portfolio optimization problem in terms of maximizing the difference between income and costs for all projects started during the planned period; to propose a method for solving the project portfolio optimization problem in terms of the social effects of projects that started during the planned period; create a method for solving the problem of project portfolio optimization for the planned period in a multi-criteria setting. There are the following **results** obtained in the paper. There is presents the mathematical model of the problem being solved, the problem objective functions include the difference between the receipt and expenditure of funds in time, the portfolio risks, and its implementation social effects. The mathematical model considers the provision of funds sufficiency for the implementation of projects in all periods, the required sequence of project implementation, and the mandatory inclusion of some projects in the portfolio for a given period. The problem under consideration belongs to the multi-criteria non-Markov dynamic discrete optimization problems. There is a proposed method for solving it in a multi-criteria formulation. The method is based on solving one criterion problem, and then a multi-criteria problem. The method is based on the minimax approach and implicit search. There has been developed solving method for the problem of enterprise project portfolio optimization for the planned period following the profit criterion. In contrast to the existing methods, this method considers the constraints on debt absence and the aftereffects of the previously made decisions. The method served as the basis for creating risk and social effect optimization methods. A method for enterprise project portfolio optimization of the planned period is provided, which, unlike previous, considers the criteria of profit, risks, and social effect, the constraints on debt absence, and the aftereffect of the previously made decisions. That makes it possible to improve the quality of the generated portfolio.*

Keywords: *project portfolio; model; multi-criteria optimization; planned period; aftereffect; method.*

Introduction

The majority of the companies carry out simultaneously not one, but a certain set of projects. Moreover, this is true not only for project-oriented companies, but also for those receiving income from operating activities. Projects of the same company often differ from each other significantly. Some projects are planned to generate income, others are aimed at solving social or environmental problems. If we consider business projects only, they also differ significantly in terms of capital investment, income expected, periods of payback, risks and many other indicators. In this regard, there arises a complicated task to form a portfolio of projects that is optimal in terms of economic indicators, social effect, and risks. It is also necessary to take into account:

– the funds sufficiency constraints for the project portfolio implementation in each considered time period,

– the constraints on some project’s implementation sequence,

– the obligation to include certain projects into the portfolio in a certain time period.

Project portfolio management is one of the managers’ in project-oriented enterprises most important functions. The efficiency of the business essentially depends on the organization of such management. There are the widely adopted standards and guidelines in this area [1 - 3].

Standard [1] proposes the portfolio life cycle, which consists of four stages: Initiation, Planning, Execution and Optimization. According to [2], the portfolio life cycle consists of the definition cycle and the delivery cycle of the portfolio.

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The authors of the paper [4] proposed the portfolio planning approach, consisting of four phases: Mapping, Simulation, Optimization, and Decision.

In the work [5], there is a framework concept of the project portfolio construction applied instead of the project portfolio selection. Authors described the multi-staged process of portfolio construction, which includes the stages of Identification, Categorization, Qualitative / Quantitative analysis, Evaluation, Prioritization, Balancing.

Authors of [6] regarded the tools and approaches to the project portfolio selection. They were classified into 4 categories: portfolio mapping tools, multi-criteria ranking tools, mathematical programming tools, and hybrid tools.

Authors of [7] executed the cluster analysis of 298 publications on the project portfolios optimization, published from 2000 to 2019. As the result, they identified 24 clusters, of which 12 clusters containing 210 papers were taken for further study. In addition, there was proposed the special cluster, dedicated to the latest publications on this topic that have already been a lot referenced to. Articles that fall into each cluster are briefly characterized.

In [8], there are regarded more than 140 works, devoted to the problem of a project portfolio choosing. The articles were classified by the criteria that are used in optimization, by the method of uncertainties accounting, by the approaches to modeling. This research highlights the papers, in which exact and heuristic methods of solution are proposed and the methods application examples are considered.

Projects portfolios optimization is carried out using various objective functions. The net present value (NPV) is often used, as are the portfolio total strategic value, potential income from portfolio implementation, standard deviations for income, costs, synergic effects from projects in the portfolio [4, 9 – 11].

Among the constraints that are taken into account when optimizing a projects portfolio, there should be accented the constraints on available resources, on the logical connections between projects, on the ratio of risks in projects, on the ratio of project categories and projects payback periods [9,10].

Optimization of project portfolios in many works was carried out without taking into account the projects start time. Among these works, the [12] is remarkable, where the authors maximized an NPV for projects portfolio taking into account the constraints on investment budget and labor resources.

The two-criteria portfolio optimization problem is considered in [13]. The first objective function maximizes the strategic gain of the portfolio. The second objective function maximizes the minimum strategic gain among sub-portfolios. The model is static.

Authors of the paper [6] denoted that one of the most important restrictions of existing approaches in the field of project portfolios optimization is the insufficient consideration of the relationships between projects inside the portfolio. The same conclusion was reached earlier by [14, 15]. Authors [6] considered 4 types of between-projects interactions: outcomes, resources consumption, success probability, and mutual including / excluding. They proposed the mathematical model of the project portfolio optimization problem with the objective function equal to the difference between outputs and inputs, and, in fact, the difference between income and costs. Using matrices, they set the success probabilities, as well as the projects' cost levels depending on the implementation of other projects. Then they proposed constraints of the project's mutual inclusion or exclusion, the number of projects in a portfolio, the income and costs in a portfolio, and on the strategic goals achieving. The proposed mathematical model is static, i.e. does not take into account the time factor.

Authors of [4] drew attention to the importance of the project's mutual influences consideration. In the optimization phase, they solved the problem of the project portfolio optimization according by the NPV and costs criteria. To do this, the binary-integer linear program was applied. The proposed project portfolio optimization was carried out without taking into account the projects' start time, i.e. in static.

The study [16] considers the interaction of projects in portfolio optimization. However, the problem is considered in statics.

The authors of a number of researches have pointed out the importance of the projects in a portfolio start time determining [9, 10]. This is significant in terms of the scarce resource's allocation.

In [17], the authors maximized the projects' effects sum total. Doing so, they also took a number of restrictions into account: the project can be started only once, the costs of all projects in a certain period should not be more than specified, projects must be completed during the planned period, and some projects must be necessarily included into the portfolio. Before the project starts, all preceding projects must be started and completed. If the certain project is included into the portfolio, then those that are not compatible with it should not be included. The model can take into account the restrictions on long-term or high-risk projects investment. The problem is related to 0-1 integer linear programming or 0-1 ILP.

Authors of [18] proposed a mathematical model for the project portfolio optimization, which is focused on NPV maximizing for the planned period. The authors use the model to define a project portfolio regarding the project start date. They suggest considering the between-projects impact using a Dependency Matrix. For each year of the planned period, there has been set a possible budget, which must not be exceeded. The model's restrictions also include the possible number of projects in the portfolio, which also should not be exceeded, and the number of projects supporting each strategic goal, which should not be less than the number specified. In general, this mathematical model is a non-linear integer model.

The authors of [19] proposed the mathematical model of the problem of the project portfolio optimization, where the objective function is equal to the NPV for the entire portfolio. The model restrictions take into account the different types of resources availability and the sequence in project operations implementation. Authors determine a portfolio considering the projects starting time.

The paper [20] suggests the problem model, in which the objective function reflects the funds balance on the organization's account at the end of the planned period. The model takes into account the restrictions on resources, on the sequence of projects implementation, on the certain project’s obligatory inclusion into the portfolio, on the level of debt in each time period, on the fact that the project can be started no more than once. In addition, the model contains a restriction that takes into account the funds availability for the project’s implementation in all periods. The resulting problem belongs to the Mixed Integer Linear Programming (MILP) problems. A robust version of this model is proposed for the case when the project’s individual stages costs are uncertain.

In [21] it was proposed to decide on the project portfolio in two phases. In the first phase, the priorities of technological areas are determined. For the decision in the second phase, a two-stage stochastic technology portfolio planning model is proposed, taking into account the risks of technological projects and the export market.

The authors of [22] proposed the mathematical model of multi-criteria project portfolio optimization for the planned period. In contrast to the known works, this model takes into account the aftereffect from the previously taken decisions. The model contains the objective functions as follows: the difference between income and expenses of a project portfolio, risks and social impact from its implementation. The model takes into account the restrictions on the fund’s availability in all years for the project portfolio implementation. Earned funds can be used for the following periods. Due to the restrictions, the interrelated projects implementation order is set, as well as the obligation to include some projects into the portfolio for a given period of time. The problem under consideration is a multi-criteria non-Markov dynamic discrete optimization problem [23].

In [24], the method for the [22] problem solving is proposed, accounting for the one objective function. There is considered the problem of the difference maximization between income and costs for all projects started during the planned period. The restrictions of the [22] problem are taken into account in full. The proposed method belongs to the methods of implicit search.

Analysis of the literature on the project portfolios optimization demonstrated that existing works did not consider the problem-solving methods that would take into account the funds receipt and expenditure dynamics in projects for the planned period and the previously made decisions aftereffect. Herewith, in problems the restrictions should be taken into account, that the funds earned in projects to a certain period have to be greater than or equal to the costs for these periods.

The present paper aims to create a method that would allow the project portfolio optimization for the planned period multi-criteria problem solving, taking into account the aftereffects of previously made decisions [22]. The objective functions of the problem include the difference between the funds receipt and expenditure over time, the portfolio risks and its implementation social effects. The mathematical model takes into account the funds sufficiency provision in all periods for the project’s implementation, the required sequence of projects implementation, the obligatory inclusion of certain projects into the portfolio at a given period of time.

1. Model of the project portfolio optimization problem for the planned period, taking into account the aftereffect of previously made decisions

The problem under consideration is the optimization of portfolio of projects that can be started on the section [1, T]. The time unit here means a period of time in relevance to which the projects in the company and the receipt of funds for them are being planned. As applied to IT projects, it is convenient to choose one week, the sprint duration (2-4 weeks) or 1 month as a time unit.

There are J projects under consideration that can potentially be included into the portfolio. The j project can be started in periods $t = \overline{1, T}$, the payment from customers can come in periods $t = \overline{1, T + l^{(j)} - 1}$, where $l^{(j)}$ is the quantity of time units, during which the works on the j-th project are done and it is financed. For the j-th project, the customer will pay c_{jr} funds in the r-th period from its start, $r = \overline{1, l^{(j)}}$.

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The j-th project expenditures will be equal to w_{jr} in the r-th period from its execution start, $r = \overline{1, l^{(j)}}$, $l^{(j)}$ – the time (quantity of units) of the project execution, when the funds may be spent on the project.

Let us assume $\max_j l^{(j)} = g$.

It is necessary to optimize the project portfolio so that in each period $t = \overline{1, T+g-1}$, $\forall j = \overline{1, J}$, there would be enough funds for its realization, the sequence of interrelated projects implementation would be kept, the project would be implemented no more than once. At the same time, it is necessary to maximize the company's profit from the project's implementation, minimize the risks associated with them, and maximize the social effect from the portfolio's projects implementation.

$$\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{r=1}^{l^{(j)}} (c_{jr} - w_{jr})\, x_{jt} \rightarrow \max, \quad (1)$$

$$\sum_{t=1}^{T} \sum_{j=1}^{J} R_j\, x_{jt} \rightarrow \min, \quad (2)$$

$$\sum_{t=1}^{T} \sum_{j=1}^{J} S_j\, x_{jt} \rightarrow \max, \quad (3)$$

$$\sum_{r=1}^{t} C_r^0 + \sum_{j=1}^{J} \sum_{p=1}^{t} \sum_{r=1}^{t-p+1} c_{jr} x_{jp} \geq \sum_{j=1}^{J} \sum_{p=1}^{t} \sum_{r=1}^{t-p+1} w_{jr} x_{jp},$$

for $t = \overline{1, T}$,

$$\sum_{r=1}^{t} C_r^0 + \sum_{j=1}^{J} \sum_{p=1}^{T} \sum_{r=1}^{t-p+1} c_{jr} x_{jp} \geq \sum_{j=1}^{J} \sum_{p=1}^{T} \sum_{r=1}^{t-p+1} w_{jr} x_{jp},$$

$$\text{for } t = \overline{T+1, T+g-1}, \quad (4)$$

$$\sum_{t=1}^{T} x_{jt} \leq 1, \;\; j = \overline{1, J}, \quad (5)$$

$$x_{jt} \cdot \operatorname{card} P_j - \sum_{p \in P_j} \sum_{m=1}^{t-l^{(p)}} x_{pm} \leq 0, \;\; t = \overline{1, T}, \quad (6)$$

$$\sum_{t=t_{s1}}^{t_{s2}} x_{st} = 1, \quad (7)$$

$$x_{jt} \in \{0,1\}, \;\; j = \overline{1, J}\,, \;\; t = \overline{1, T}, \quad (8)$$

where R_j stands for the risks, related with the j-th project implementation,

S_j – the social effect of the j-th project,

C_r^0, $r = \overline{1, T+g-1}$, – the funds, which the company can allocate for the project portfolio implementation during the r period. These funds value can also be negative if the company in the r period needs to receive funds through the project portfolio implementation,

x_{jt} – the Boolean variable, equal to 1, if the j-th project is started in t-th year, and equal to 0 if opposite.

For the j project, the set of project numbers P_j can be defined for the projects that are to be implemented before the j project starts.

The objective function of (1) problem represents the difference between income and costs for all projects starting from the first period to T-th. As the j-th project is being implemented during $l^{(j)}$ periods, $j = \overline{1, J}$, the objective function (1) takes into account the income and costs of projects started in the period from the first to T-th, and also after T period, during $l^{(j)} - 1$ periods, $\forall j = \overline{1, J}$.

The objective function (2) represents the risks associated with the portfolio implementation. A list of the most significant risks for each project is compiled. Risks are assessed by multiplying the risk event occurrence probability by its consequences. The consequences of a risk event are evaluated in points from 0 to 10 in accordance with the table 1.

Table 1

Assessment of the risk events consequences

Negative consequences	Points
Impacts that lead to the project termination or complete failure	10
Impacts that lead to very significant project delays, over budgeting, and degraded project product quality	8-9
Impacts that lead to significant project delays, over budgeting, and degraded project product quality	6-7
Impacts that lead to the not very significant project delays, budget overruns, the project product quality deterioration	4-5
Impacts that lead to insignificant project delays, over budgeting, and project product quality deterioration	2-3
Negative consequences are almost inappreciable	1
Negative consequences are absent	0

The risk assessment for the j-th project equals:

$$R_j = \sum_{k=1}^{K^{(j)}} \alpha_{kj} e_{kj},$$

where α_{kj} represents the probability of the k-th risk event occurrence for the j-th project,

e_{kj} – the negative consequences (in points) from the k-th risk event for the j-th project,

$K^{(j)}$ – risk event quantity for the j-th project.

The (3) objective function represents the social effect of projects started during the planned period. The social effect of the j-th project implementation may consist in the personnel qualifications improvement as a result of this project implementation, increasing the personnel's salaries, in solving the team's or community social problems related to this project. The social effect can be expressed in points, as shown in the table 2.

Table 2

Project social effects assessment

Social effects	Points
Very significant social effect	9-10
Significant social effect	7-8
Average social effect	5-6
Insignificant social effect	3-4
The social effect is barely noticeable	1-2
No social effect	0

The restriction (4) demands, that in t period, $t = \overline{1, T+g-1}$, the funds earned by projects started in and to the t period, were more or equal to the expenditures for these periods.

The restriction (4) for $t = \overline{1,T}$ applies to the periods, when there can be started the projects included into the portfolio.

The restriction (4) for $t = \overline{T+1, T+g-1}$, applies to the periods after T, in which there continues the implementation of projects started before T and during T, inclusively.

The first term on the left side of the constraint (4), as for $t = \overline{1,T}$, as for $t = \overline{T+1, T+g-1}$ alike, i.e.

$$F_1 = \sum_{r=1}^{t} C_r^0;$$

– these are the accumulated funds by *t* period and in *t* period, that the company can allocate for the projects portfolio implementation.

The second term on the left side of the constraint (4) for $t = \overline{1,T}$, i.e.

$$F_2 = \sum_{j=1}^{J} \sum_{p=1}^{t} \sum_{r=1}^{t-p+1} c_{jr} x_{jp} \;;$$

– this is the accumulated income for *t* period and in *t* period from projects that were started in periods from 1st to *t*, inclusively.

The right side of the constraint (4) for $t = \overline{1,T}$

$$F_3 = \sum_{j=1}^{J} \sum_{p=1}^{t} \sum_{r=1}^{t-p+1} w_{jr} x_{jp};$$

– these are the accumulated costs for t period and in t period from projects that were started in periods from 1st to T, inclusively.

The second term on the left side of the constraint (4) for $t = \overline{T+1, T+g-1}$, i.e.

$$F_4 = \sum_{j=1}^{J} \sum_{p=1}^{T} \sum_{r=1}^{t-p+1} c_{jr} x_{jp} \;;$$

– this is the accumulated income for *t* period and in *t* period from projects that were started in periods from 1st to *t*, inclusively.

The right side of the (4) constraint for $t = \overline{T+1, T+g-1}$, i.e.

$$F_5 = \sum_{j=1}^{J} \sum_{p=1}^{T} \sum_{r=1}^{t-p+1} w_{jr} x_{jp};$$

– these are the accumulated costs for t period and in t period from projects that were started in periods from 1st to T, inclusively.

The constraint (5) demands any j-th project, $j = \overline{1, J}$, to be implemented no more than once.

The constraint (6) assumes that before the start of the j-th project, projects from the P_j set should be implemented. The second term in (6) is equal to the sum of units, each of which corresponds to a project from the P_j set, implemented before the current time t.

The constraint (7) allows setting the demand of the obligatory s-th project inclusion into the portfolio on the time interval $[t_{s1}, t_{s2}]$.

The problem (1) - (8) belongs to the multi-criteria dynamic Boolean programming problems. It is also possible to characterize problem (1) - (8) as a multi-criteria non-Markov dynamic problem of the discrete optimization [23]. Non-Markov are called those optimization problems in which the state of the object at the t-th stage is a function of the state at the previous stage t-1 and controls at the stages t, t-1, t-2, …, t-p+1. I.e., the aftereffect of the controls applied earlier is taken into account. So for the problem of project portfolio optimization, the decision to start a certain j-th project with duration of $l^{(j)}$ periods in the t period will affect the state of the portfolio during t+1, t+2,…, $t+l^{(j)} - 1$ periods.

2. Transformation of the problem objective functions

Consider a method for solving problem (1) - (8). In order to simplify the essence of the method, the restriction (6) and (7) will not be taken into account.

Let us transform the objective functions (1) - (3) of problem (1) - (8) to convert them to dimensionless form

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and, in addition, to make necessary to minimize all objective functions after the transformation.

For the transformation, the monotone functions of the following form will be used [25]:

$$A^n(x_{jt}) = \frac{A^{opt}(x_{jt}) - A(x_{jt})}{A^{opt}(x_{jt}) - A^{min}(x_{jt})}; \quad (9)$$

$$R^n(x_{jt}) = \frac{R(x_{jt}) - R^{opt}(x_{jt})}{R^{max}(x_{jt}) - R^{opt}(x_{jt})}; \quad (10)$$

$$S^n(x_{jt}) = \frac{S^{opt}(x_{jt}) - S(x_{jt})}{S^{opt}(x_{jt}) - S^{min}(x_{jt})}; \quad (11)$$

where $A(x_{jt})$, $R(x_{jt})$, $S(x_{jt})$ – the (1) (2) and (3) objective functions values, $A^{min}(x_{jt})$, $S^{min}(x_{jt})$ – the (1) and (3) objective functions lowest values, which are attained on the set of admissible alternatives, $R^{max}(x_{jt})$ – the (2) objective function maximum value, which is attained on the set of admissible alternatives, $A^{opt}(x_{jt})$, $R^{opt}(x_{jt})$, $S^{opt}(x_{jt})$ – the (1) – (3) objective functions optimal values that are attained on the set of admissible alternatives.

As a result of the transformation, $A^n(x_{jt})$, $R^n(x_{jt})$, $S^n(x_{jt})$ values will vary in the range from 0 to 1.

To find worst-case estimates $A^{min}(x_{jt})$, $R^{max}(x_{jt})$, $S^{min}(x_{jt})$ of the (1) - (8) problem, two types of algorithm can be proposed. Each of them does not guarantee finding the really worst solutions of (1), (4), (5), (8); (2), (4), (5), (8); (3), (4), (5), (8) problems, satisfying the constraints, but allows to find solutions not knowingly better than $A^{opt}(x_{jt})$, $R^{opt}(x_{jt})$, $S^{opt}(x_{jt})$.

The first type algorithms actually boil down to the equaling the $A^{min}(x_{jt})$, $R^{max}(x_{jt})$, $S^{min}(x_{jt})$ to the first admissible solution that is obtained in the process of solving problems (1), (4), (5), (8); (2), (4), (5), (8); (3), (4), (5), (8), respectively. This approach has a significant drawback: if during the optimization process a better solution than the first admissible one is not found, then 0 will appear in the denominator of the (9), (10), (11) formulas. In this case, the objective functions normalized values would not be calculable.

Algorithms of the second type involve the following actions.

For evaluation $A^{min}(x_{jt})$, we should rank the values of target functions (1) for each of the considered projects, i.e. $\sum_{t=1}^{T}\sum_{r=1}^{l^{(j)}}(c_{jr} - w_{jr})$, $j = \overline{1,J}$, and select T projects with the lowest values of target functions (1) that are less than zero. If such values are less than T, we take as many as there are. The sum of the selected values of the target functions can be used as $A^{min}(x_{jt})$. If there are no such values, $A^{min}(x_{jt})=0$.

Estimation $R^{max}(x_{jt})$ can be done by ranking R_j for all projects in the portfolio under consideration. Then the T highest values of R_j. are selected. If such values are less than T, we take as many as there are. The sum of the selected values of the target functions can be used as $R^{max}(x_{jt})$. If there are no such values, this indicates that risk criterion (2) is not used in the analyzed projects. Optimization according to this criterion does not need to be carried out.

It is possible to take $S^{min}(x_{jt})=0$ as an estimate $S^{min}(x_{jt})$. I.e., the minimum social effect is achieved in the absence of projects that produce such an effect.

The flaw of this approach lies in possibility that selected projects may not satisfy the problem constraints. As a result, estimates for $A^{min}(x_{jt})$, $S^{min}(x_{jt})$ will be lowered, and the estimation of $R^{max}(x_{jt})$ will be raised.

3. The method of the project portfolio optimization for the planned period in terms of profit

Let us consider the method of the $A^{opt}(x_{jt})$ finding. We assume that only one project can be started in t period, $t = \overline{1,T}$. The method will be presented for the case of the objective function minimization; therefore the $A(x_{jt})$ objective function will be negated.

1. Determine the J – the number of projects that can be included in the portfolio and the T period, for which the portfolio will be formed. Set the initial values of the problem parameters. $\overline{W_T} = \varnothing$.

Set the income c_{jr} and costs w_{jr} by each project, and also each project duration $l^{(j)}$, $j = \overline{1,J}$, $r = \overline{1,l^{(j)}}$.

Determine the funds C_k^0, which the company can allocate for the project portfolio implementation during the period k, $k = \overline{1,T+g-1}$. We set g – the maximum possible time (number of periods) for the project implementation.

Set the $f = 0, t' = 1$, where t' is the current time period. Assign the record objective function value $f^* := \infty$.

2. Check the ability not to start projects in the period t', i.e. $j(t')=0$.

3. If $t'<T$ and $F_1+F_2\ge F_3$ for $t=\overline{1,t'}$, then move to the step 4.3, where $F_1=\sum_{k=1}^{t}C_k^0$, $F_2=\sum_{k=1}^{t}M_k$, (here $M_k=c_{j(k),1}+c_{j(k),2}+...+c_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$), $F_3=\sum_{k=1}^{t}M_k$, (here $M_k=w_{j(k),1}+w_{j(k),2}+...+w_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$).

If $t'=T$ and $F_1+F_2\ge F_3$ for $t=\overline{1,T}$, $F_1+F_4\ge F_5$, for $t=\overline{T+1,T+g-1}$, move to the step 4.3, where $F_1=\sum_{k=1}^{t}C_k^0$, $F_4=\sum_{k=1}^{T}M_k$, (here $M_k=c_{j(k),1}+c_{j(k),2}+...+c_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$), $F_5=\sum_{k=1}^{T}M_k$, (here $M_k=w_{j(k),1}+w_{j(k),2}+...+w_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$).

Otherwise, consider the first project, i.e. $j(t'):=j(t')+1$.

4. Carry out the actions as follows.

4.1 Check the (4) constraint fulfillment.

If $t'<T$ and $F_1+F_2\ge F_3$ for $t=\overline{1,t'}$, where $F_1=\sum_{k=1}^{t}C_k^0$, $F_2=\sum_{k=1}^{t}M_k$, (here $M_k=c_{j(k),1}+c_{j(k),2}+...+c_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$), $F_3=\sum_{k=1}^{t}M_k$, (here $M_k=w_{j(k),1}+w_{j(k),2}+...+w_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$) or, if $t'=T$ and $F_1+F_2\ge F_3$ for $t=\overline{1,T}$, $F_1+F_4\ge F_5$, for $t=\overline{T+1,T+g-1}$, where $F_1=\sum_{k=1}^{t}C_k^0$, $F_4=\sum_{k=1}^{T}M_k$, (here $M_k=c_{j(k),1}+c_{j(k),2}+...+c_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$), $F_5=\sum_{k=1}^{T}M_k$, (here $M_k=w_{j(k),1}+w_{j(k),2}+...+w_{j(k),p}$, $p=\min\{t-k+1,l^{(j(k))}\}$, if $j(k)\neq 0$; $M_k=0$, if $j(k)=0$), move to the step 4.2.

Otherwise, move to the step 7.

4.2 If $t'>1$, check the fulfillment of the (5) constraint for the j-th project

$j(t')\neq j(t), t=\overline{1,t'-1}$.

If it isn't fulfilled, move to the step 7.

4.3 If $t'=T$, then $D^{(t')U}:=0$, move to the step 4.4. Otherwise, determine the lower bound for the problem objective function for the $T-t'$ period. The search for the lower limit is carried out by evaluating the maximum profit that can be obtained from the projects implementation, from those that have not been started yet. The problem constraints are not taken into account here. I.e., consider all $j\in B_{t'}$, where $B_{t'}=B\setminus W_{t'}$, $B=\{1,2,...,J\}$ – set of projects' indices (numbers) that are considered in the problem, $W_{t'}=\{j(i), i=\overline{1,t'}\}$ – set of projects' numbers that are assigned in periods from 1st to t'-th.

Calculate

$$A_j^{(t')}=\sum_{r=1}^{l^{(j)}}(c_{jr}-w_{jr})\quad \forall j\in B_{t'}.$$

Rank $A_j^{(t')}$ from largest to smallest. From the ranked values, we select $T-t'$ the highest values that are greater than zero. If there are less ranked values $A_j^{(t')}>0$, than $T-t'$, take as much as there is. From the numbers of the selected projects, form a set $E_{t'}$. Then calculate $D^{(t')}=\sum_{\forall j\in E_{t'}}A_j$ and assign $D^{(t')U}:=D^{(t')}$. The latter assignment is explained by the fact that if future options are unprofitable, then there is an opportunity to do nothing.

If the ranked values set $A_j^{(t')}>0\ \forall j\in B_{t'}$ is empty, then assign $D^{(t')U}:=0$.

4.4 At $j(t')=0$ and $f-D^{(t')U}\ge f^*$ an attempt to not start a project in a year t' does not allow finding a solution better than the record one. Move to the step 7.

At $j(t')>0$ and at $f-\sum_{r=1}^{l^{(j)}}(c_{jr}-w_{jr})-D^{(t')U}\ge f^*$ the j-th project start in t'-th year doesn't allow finding the solution better than the record one. Move to the step 7.

4.5 At $j(t')>0$ assign $f:=f-\sum_{r=1}^{l^{(j)}}(c_{jr}-w_{jr})$.

5. If $t'<T$, then consider the next year, $t':=t'+1$, move to the step 2.

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6. Diminish the record value $f^* := f$. Remember the set $\overline{W_T} = \{j(t), t = \overline{1,T}\}$. If $j(T) \neq 0$, make an assignment $f := f + \sum_{r=1}^{l^{(j(T))}} (c_{j(T)r} - w_{j(T)r})$.

7. At $j(t') < J$ assign $j(t') := j(t') + 1$ and move to the step 4.

8. At $t' > 1$ jump one year before, i.e. $t' := t' - 1$. At $j(t') \neq 0$ change the value of $f := f + \sum_{r=1}^{l^{(j(t'))}} (c_{j(t')r} - w_{j(t')r})$. Move to the step 7.

If $t' = 1$, and $\overline{W_T}$ = Ø, the problem under consideration has no solution, otherwise the optimal project portfolio for the planned period is obtained.

At $\overline{W_T} = \{0, 0, ..., 0\}$ the optimal solution is not to implement projects in the planned period. In this case, the implementation of any project is either not acceptable in terms of constraints, or is no better than no projects at all.

The proposed method refers to implicit enumeration methods. It differs from the existing ones by taking into account the non-Markov nature of the problem being solved, i.e. takes into account the aftereffects of previously made decisions.

4. The method of the project portfolio optimization for the planned period in terms of risks

Consider the method of finding the $R^{opt}(x_{jt})$. We describe the method for the case of the objective function minimization. It should be noted that if there are no projects that must be started in the planned period, then the minimum of the (2) objective function, i.e. a minimum of risks, will be achieved in the absence of projects starting in the planned period. This is due to the fact that, in this formulation, risks are inherent only to the projects. If there are no projects, then there are no risks. The need to solve the problem of finding $R^{opt}(x_{jt})$ appears, if the organization has set at least one negative value of C_r^0, $r = \overline{1, T + g - 1}$, – i.e., of the funds, which the company can allocate for the projects portfolio implementation during the r period.

1. Determine the J – the number of projects that can be included into the portfolio and the T period, for which the portfolio will be formed. Set the initial values of the problem parameters. $\overline{W_T}$ = Ø.

Determine the income c_{jr} and costs w_{jr} by each project, R_j – the risk, related to the j-th project implementation, and also each project duration $l^{(j)}$, $j = \overline{1,J}$, $r = \overline{1, l^{(j)}}$.

Determine the funds C_k^0, which the company can allocate for the project portfolio implementation during the period k, $k = \overline{1, T + g - 1}$. Set the maximum able time g (number of periods) of the project implementation.

Assign the $f = 0, t' = 1$, where t' is the current time period. Assign the record value of the objective function $f^* := \infty$.

Steps 2, 3, as well as 4.1, 4.2 of the present method repeat similar steps of the optimization method $A(x_{jt})$.

4.3 If $t' = T$, then $D^{(t')U} := 0$, move to the step 4.4. Otherwise, we determine the lower bound for the problem objective function for the period $T - t'$. Since the minimum risk in this formulation of the problem is achieved in the absence of projects with risk, then $D^{(t')U} := 0$.

4.4 At $j(t') = 0$ и $f + D^{(t')U} \geq f^*$ attempt to not start a project in a year t' does not allow finding a solution better than the record one. Move to the step 7. At $j(t') > 0$ and at $f + R_j + D^{(t')U} \geq f^*$ the j-th project start in the t'-th year does not allow finding a solution better than the record one. Move to the step 7.

4.5 At $j(t') > 0$ assign $f := f + R_j$.

5. If $t' < T$, then consider the next year, $t' := t' + 1$, move to the step 2.

6. Diminish the record value $f^* := f$. Remember the set $\overline{W_T}$=$\{j(t),\ t = \overline{1,T}\}$. If $j(T) \neq 0$, make an assignment $f := f - R_j$.

7. At $j(t') < J$ assign $j(t') := j(t') + 1$ and move to the step 4.

8. At $t' > 1$ jump one year before, i.e., $t' := t' - 1$. At $j(t') \neq 0$ change the value $f := f - R_j$. Move to the step 7.

If $t' = 1$, and $\overline{W_T}$ =Ø, the problem under consideration has no solution, otherwise the optimal project portfolio for the planned period is obtained.

At $\overline{W} = \{0, 0, \ldots, 0\}$ the optimal solution is not to implement projects in the planned period.

5. The method of project portfolio optimization for the planned period in terms of social effect

Consider the method of finding the $S^{opt}(x_{jt})$. We describe the method for the case of the objective function minimization. Due to this, the objective function $S(x_{jt})$ will be taken negated.

1. Determine the J – the number of projects that can be included into the portfolio and the T period, for which the portfolio will be formed. Set the initial values of the problem parameters. $\overline{W_T} = \varnothing$.

Determine the income c_{jr} and costs w_{jr} by each project, S_j – the social effect, related to each project implementation, and also each project duration $l^{(j)}$, $j = \overline{1,J}$, $r = \overline{1,l^{(j)}}$.

Determine the funds C_k^0, which the company can allocate for the projects portfolio implementation during the period k, $k = \overline{1, T+g-1}$. Set the maximum able time g (number of periods) of the project implementation.

Assign the $f = 0, t' = 1$, where t' is the current time period. Assign the record value of the objective function $f^* := \infty$.

Steps 2, 3, as well as 4.1, 4.2 of the present method repeat similar steps of the optimization method $A(x_{jt})$.

4.3 If $t' = T$, then $D^{(t')U} := 0$, move to the step 4.4. Otherwise, we determine the lower bound for the problem objective function for the period $T - t'$. To do this, assess the maximum possible social effect that can be obtained from the implementation of projects that have not been started yet. The problem constraints are not taken into account here. I.e., consider all $j \in B_{t'}$, where $B_{t'} = B \backslash W_{t'}$, $B = \{1,2,\ldots,J\}$ – the set of projects' indices (numbers) that are considered in the problem, $W_{t'} = \{j(i), i = \overline{1,t'}\}$ – the set of projects' numbers that are assigned in the periods from 1st to t'th.

Consider S_j $\forall j \in B_{t'}$.

Rank S_j from largest to smallest. Choose from the ranked $T-t'$ of the highest values. If there are less ranked values, than $T-t'$, take as much as there is. From the numbers of the selected projects, form a set $E_{t'}$. Calculate $D^{(t')} = \sum_{\forall j \in E_{t'}} S_j$. $D^{(t')U} := D^{(t')}$.

If the set of ranked values S_j $\forall j \in B_{t'}$ is empty, then assign $D^{(t')U} := 0$.

4.4 At $j(t') = 0$ and $f - D^{(t')U} \geq f^*$ attempt to not start a project in a year t' does not allow finding a solution better than the record one. Move to the step 7. At $j(t') > 0$ and at the $f - S_j - D^{(t')U} \geq f^*$ the j-th project start in the t'-th year does not allow finding a solution better than the record one. Move to the step 7.

4.5 At $j(t') > 0$ assign $f := f - S_j$.

5 If $t' < T$, then consider the next year, $t' := t'+1$, move to the step 2.

6 Diminish the record value $f^* := f$. Remember the set $\overline{W_T} = \{j(t), t = \overline{1,T}\}$. If $j(T) \neq 0$, make an assignment $f := f + S_j$.

7 At $j(t') < J$ assign $j(t') := j(t') + 1$ and move to the step 4.

8 At $t' > 1$ jump one year before, i.e., $t' := t' - 1$. At $j(t') \neq 0$ change value $f := f + S_j$. Move to the step 7.

If $t' = 1$, and $\overline{W_T} = \varnothing$, the problem under consideration has no solution, otherwise the optimal project portfolio for the planned period is obtained.

At $\overline{W_T} = \{0, 0, \ldots, 0\}$ the optimal solution is not to implement projects in the planned period.

6. The method of multi-criteria project portfolio optimization for the planned period

Let us turn to the presentation of the method for solving the multi-criteria problem (1) - (5), (8). To solve this problem, it is proposed to use the minimax approach. Among the admissible problem solutions, it is necessary to find the one that minimizes the maximum deviations from the optimal solutions of one-criterion problems (1), (4), (5), (8); (2), (4), (5), (8); (3), (4), (5), (8), respectively.

Consider the method for solving the multi-criteria problem (1) - (5), (8).

1. Determine the J – the number of projects that can be included into the portfolio and the T period, for which the portfolio will be formed. Set the initial values of the problem parameters. $\overline{W_T} = \varnothing$.

Determine the income c_{jr} and costs w_{jr} by each project, R_j – risks, related to the j-th project implementation, S_j – the social effect, related to each project implementation, and also each project duration $l^{(j)}$, $j = \overline{1,J}$, $r = \overline{1,l^{(j)}}$.

Determine the funds C_k^0, which the company can allocate for the projects portfolio implementation during the period k, $k = \overline{1, T+g-1}$. Set the maximum able time g (number of periods) of the project implementation.

Assign $f^A = 0$, $f^R = 0, f^S = 0$, $t' = 1$, where t' is the current time period. Assign the record value of the objective function $f^* := \infty$.

2. Check the ability not to start projects in the period t', i.e. $\mathbf{j(t') = 0}$.

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3. If $t' < T$ and $F_1 + F_2 \ge F_3$ for $t = \overline{1, t'}$, move to the step 4.3, where $F_1 = \sum_{k=1}^{t} C_k^0$, $F_2 = \sum_{k=1}^{t} M_k$, (here $M_k = c_{j(k),1} + c_{j(k),2} + ... + c_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$), $F_3 = \sum_{k=1}^{t} M_k$, (here $M_k = w_{j(k),1} + w_{j(k),2} + ... + w_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$).

If $t' = T$ and $F_1 + F_2 \ge F_3$ for $t = \overline{1, T}$, $F_1 + F_4 \ge F_5$, for $t = \overline{T+1, T+g-1}$, move to the step 4.3, where $F_1 = \sum_{k=1}^{t} C_k^0$, $F_4 = \sum_{k=1}^{T} M_k$, (here $M_k = c_{j(k),1} + c_{j(k),2} + ... + c_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$), $F_5 = \sum_{k=1}^{T} M_k$, (here $M_k = w_{j(k),1} + w_{j(k),2} + ... + w_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$).

Otherwise, consider the first project, i.e. $j(t') := j(t') + 1$.

4. Carry out the actions as follows.

4.1 Check the (4) constraint fulfillment.

If $t' < T$ and $F_1 + F_2 \ge F_3$ for $t = \overline{1, t'}$, where $F_1 = \sum_{k=1}^{t} C_k^0$, $F_2 = \sum_{k=1}^{t} M_k$, (here $M_k = c_{j(k),1} + c_{j(k),2} + ... + c_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$), $F_3 = \sum_{k=1}^{t} M_k$, (here $M_k = w_{j(k),1} + w_{j(k),2} + ... + w_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$) or, if $t' = T$ and $F_1 + F_2 \ge F_3$ for $t = \overline{1, T}$, $F_1 + F_4 \ge F_5$, for $t = \overline{T+1, T+g-1}$, where $F_1 = \sum_{k=1}^{t} C_k^0$, $F_4 = \sum_{k=1}^{T} M_k$, (here $M_k = c_{j(k),1} + c_{j(k),2} + ... + c_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$), $F_5 = \sum_{k=1}^{T} M_k$, (here $M_k = w_{j(k),1} + w_{j(k),2} + ... + w_{j(k),p}$, $p = \min\{t - k + 1, l^{(j(k))}\}$, if $j(k) \ne 0$; $M_k = 0$, if $j(k) = 0$), move to the step 4.2.

Otherwise move to the step 7.

4.2 If $t' > 1$, check the constraint (5) fulfillment for the j-th project $j(t') \ne j(t), t = \overline{1, t'-1}$. If it doesn't fulfill, then move to step 7.

4.3 If $t' = T$, then $D^{(t'A)} := 0$, move to the step 4.4. Otherwise, determine the lower bound for the problem objective function (1) of the problem (1) - (5), (8) for the $T - t'$ period. The search for the lower limit is carried out by evaluating the maximum profit that can be obtained from the projects implementation, from those that have not been started yet. The problem constraints are not taken into account here. I.e., consider all $j \in B_{t'}$, where $B_{t'} = B \setminus W_{t'}$, $B = \{1, 2, ..., J\}$ – the set of projects' indices (numbers) that are considered in the problem, $W_{t'} = \{j(i), i = \overline{1, t'}\}$ – the set of projects' numbers that are assigned in the periods from 1st to t′-th.

Calculate

$$A_j^{(t')} = \sum_{r=1}^{l^{(j)}} (c_{jr} - w_{jr}) \quad \forall j \in B_{t'}.$$

Rank $A_j^{(t')}$ from largest to smallest. From the ranked values, we select $T - t'$ the highest values that are greater than zero. If there are less ranked values $A_j^{(t')} > 0$, than $T - t'$, take as much as there is. From the numbers of the selected projects, form a set $E_{t'}$. Then calculate $D^{(t')} = \sum_{\forall j \in E_{t'}} A_j$ and assign $D^{(t'A)} := D^{(t')}$. The latter assignment is explained by the fact that if future options are unprofitable, then there is an opportunity to do nothing.

If the ranked values set $A_j^{(t')} > 0\ \forall j \in B_{t'}$ is empty, then assign $D^{(t'A)} := 0$.

4.4 If $t' = T$, then $D^{(t'R)} := 0$, move to the step 4.5. Otherwise, determine the lower bound for the objective function (2) for problem (1) - (5), (8) for the $T - t'$ period. To do this, assess the minimum possible risk that can be obtained from the implementation of projects that have not been started yet. The problem constraints are not taken into account here. Since in this formulation of the problem the minimum risk is achieved in the absence of projects with risk, then $D^{(t'R)} := 0$.

4.5 If $t' = T$, then $D^{(t'S)} := 0$, move to the step 4.6. Otherwise, determine the lower bound for the objective function (3) for problem (1) - (5), (8) for the $T - t'$ period. To do this, assess the maximum possible social effect that can be obtained from the implementation of projects that have not been started yet. The problem constraints are not taken into account here. I.e., consider all $j \in B_{t'}$, where $B_{t'} = B \setminus W_{t'}$, $B = \{1, 2, \dots, J\}$ - the set of projects' indices (numbers) that are considered in the problem, $W_{t'} = \{j(i), i = \overline{1,t'}\}$ – the set of projects' numbers that are assigned in periods from the 1st to t′-th.

Consider S_j $\forall j \in B_{t'}$.

Rank S_j from largest to smallest. Choose from the ranked $T-t'$ of the highest values. If there are less ranked values, than $T-t'$, take as much as there is. From the numbers of the selected projects, form a $E_{t'}$ set. Calculate $D^{(t')} = \sum_{\forall j \in E_{t'}} S_j$. $D^{(t'S)} := D^{(t')}$.

If the set of ranked values S_j $\forall j \in B_{t'}$ is empty, then assign $D^{(t'S)} := 0$.

4.6 At $j(t') = 0$

$$A(x_{jt}) = -(f^A - D^{(t'A)}),$$
$$R(x_{jt}) = f^R + D^{(t'R)},$$
$$S(x_{jt}) = -(f^S - D^{(t'S)}).$$

If $\max\{A^n(x_{jt}), R^n(x_{jt}), S^n(x_{jt})\} \geq f^*$, attempt to not start a project in a year t' does not allow finding a solution better than the record one. Move to the step 7.

At $j(t') > 0$ calculate the value of the lower bound for each of the criteria

$$A(x_{jt}) = -(f^A - \sum_{r=1}^{l^{(j)}}(c_{jr} - w_{jr}) - D^{(t'A)}),$$
$$R(x_{jt}) = f^R + R_j + D^{(t'R)},$$
$$S(x_{jt}) = -(f^S - S_j - D^{(t'S)}).$$

At $\max\{A^n(x_{jt}), R^n(x_{jt}), S^n(x_{jt})\} \geq f^*$the start of the j-th project in t′-th year does not allow finding a solution that would be better than the record one. Move to the step 7.

4.7 Assign $f^A := f^A - \sum_{r=1}^{l^{(j)}}(c_{jr} - w_{jr})$, $f^R := f^R + R_j$, $f^S := f^S - S_j$.

5. If $t' < T$, then consider the next year, $t' := t' + 1$, move to the step 2.

6. Calculate the objective function value $f = \max\{A^n(x_{jt}), R^n(x_{jt}), S^n(x_{jt})\}$. Diminish the record value $f^* := f$. Remember the set $\overline{W_T}$=$\{j(t),\ t = \overline{1,T}\}$. If $j(t') \neq 0$, make an assignment $f^A := f^A + \sum_{r=1}^{l^{(j)}}(c_{jr} - w_{jr})$, $f^R := f^R - R_j$, $f^S := f^S + S_j$.

7. At $j(t') < J$ assign $j(t') := j(t') + 1$ and move to the step 4.

8. At $t' > 1$ jump to the previous year, i.e. $t' := t' - 1$. At $j(t') \neq 0$ change values $f^A := f^A + \sum_{r=1}^{l^{(j(t'))}}(c_{j(t')r} - w_{j(t')r})$, $f^R := f^R - R_{j(t')}$, $f^S := f^S + S_{j(t')}$. Move to the step 7.

If $t' = 1$, and $\overline{W_T} = \emptyset$, the problem under consideration has no solution, otherwise the optimal project portfolio for the planned period is obtained.

At $\overline{W} = \{0,0,\dots,0\}$ the optimal solution is not to implement projects in the planned period. This demonstrates that the implementation of any project is either not acceptable in terms of constraints or is no better than no projects at all.

Solving the problem of project portfolio optimization by three criteria according to the proposed method involves the following steps:

1. Gathering information about possible portfolio projects.
2. Determining the composition and parameters of the constraints of the problem.
3. Solving single-criteria problems of project portfolio optimization in terms of profit, risks, social effect, by the proposed methods.
4. Determination of evaluations of the worst values of the target functions $A^{min}(x_{jt})$, $R^{max}(x_{jt})$, $S^{min}(x_{jt})$.
5. The solution of the three-criteria problem of project portfolio optimization.

7. An example of the project portfolio optimization problem

Let us consider an illustrative example of solving the (1), (4), (5), (8) one-criterion problem that illustrates the aftereffects of previously made decisions. The possibility of starting no more than two projects during the planned period T=3 is being analyzed. The income from the first project will be $c_{11} = 100$, $c_{12} = 100$, $c_{13} = 30$. The costs for the first project will be $w_{11} = 50$, $w_{12} = 30$, $w_{13} = 100$. The income from the second project will be $c_{21} = 50$, $c_{22} = 100$. The costs for the second project will be equal $w_{21} = 100$, $w_{22} = 100$.

When searching all possible portfolio options, the objective function values will be obtained, that are presented in Table 3. In the second column of the table, there are indicated problem variables equal to 1, while the remaining variables are equal to 0. Line 16 shows the option to not start any projects.

Constraints (4) require that in the period of possible implementation of the projects, i.e. from the first to the fifth years, the funds earned in the projects should be greater than or equal to the costs for these years. For options №4-№6, №8, №9, №12, these restrictions are not met. Forbidden variants of the project start more than once in the table are not considered. The peculiarity of the proposed optimization method is that no more than one project can be started in one year. As a result, options №7, №11, №15 are also prohibited. The maximum of the target function is reached for variants №1, №2, №3.

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Thus, the optimal solution to the problem is to implement only the first project, which can be started in any year of the planning period. This example was used when testing software that implements the proposed method.

Conclusion

The works in the field of project portfolios optimization have been analyzed. The goal of the article has been formulated, aimed at creation of method that would allow solving the multi-criteria project portfolio optimization problem for the planned period, taking into account the aftereffects of previously made decisions. The problem objective functions include the difference between the funds receipt and expenditure in the portfolio, the risks and the social effects associated with the project portfolio implementation. It takes into account the provision of funds sufficiency in all periods for the projects implementation, a restriction on the sequence of projects implementation, the obligatory inclusion of certain projects in the portfolio in a given time period. The problem under consideration belongs to non-Markov dynamic problems of discrete optimization. The method for solving it in a multi-

Table 3

Options of solution based on constraints (4), (5), (8)

№	Problem variables equal to 1	The funds availability by year					The objective function value	Options meeting the constraints (4), (5), (8).
		Years						
		1	2	3	4	5		
1	x_{11}	50	120	50	50	50	50	✓
2	x_{12}	0	50	120	50	50	50	✓
3	x_{13}	0	0	50	120	50	50	✓
4	x_{21}	-50	-50	-50	-50	-50	-50	-
5	x_{22}	0	-50	-50	-50	-50	-50	-
6	x_{23}	0	0	-50	-50	-50	-50	
7	x_{11} x_{21}	-	-	-	-	-	-	It is forbidden to start two projects in the same year
8	x_{12} x_{21}	-50	0	70	0	0	0	-
9	x_{13} x_{21}	-50	-50	0	70	0	0	-
10	x_{11} x_{22}	50	70	0	0	0	0	✓
11	x_{12} x_{22}	-	-	-	-	-	-	It is forbidden to start two projects in the same year
12	x_{13} x_{22}	0	-50	0	70	0	0	-
13	x_{11} x_{23}	50	70	0	0	0	0	✓
14	x_{12} x_{23}	0	50	70	0	0	0	✓
15	x_{13} x_{23}	-	-	-	-	-	-	It is forbidden to start two projects in the same year
16	-	0	0	0	0	0	0	✓

criteria formulation is proposed. The method is based on solving one-criterion problems, and then a multi-criteria problem. The method is based on the minimax approach and implicit search.

The strength of the proposed method for solving the multi-criteria problem of project portfolio optimization, as well as methods for solving single criteria problems is to take into account the dynamics of receipt and expenditure of funds, the ability to require the absence of indebtedness in all periods of the projects. The methods allow taking into account a wide range of constraints, both analytical and algorithmic, including those in the form of simulation models, in the optimization model. The restriction related to the admissibility of starting not more than one project in a specific time period can be circumvented by splitting the time period into sub-periods. In this case, the restriction will apply to each sub-period. It is advisable to apply the method to optimize the project portfolio after project management approaches have been defined for potential projects and the costs of these projects have been estimated [26].

An example is given that illustrates the projects portfolio optimization problem with regard of the aftereffects of previously made decisions.

In the future, it is planned to create the software that will implement the proposed methods, and by this solve test and real problems of project portfolio optimization.

Contributions of authors: literature analysis – **A. Korchakova**; statement of the research problem – **I. Kononenko**, **A. Korchakova**; task model – **I. Kononenko**, **A. Korchakova**; the methods of the project portfolio optimization for the planned period in terms of profit, risks, and social effect – **I. Kononenko**, **A. Korchakova**; the method of multi-criteria project portfolio optimization for the planned period – **I. Kononenko**, **A. Korchakova**; an example of the project portfolio optimization problem – **A. Korchakova**; conclusion – **I. Kononenko**, **A. Korchakova**. All authors have read and agreed to the published version of the manuscript.

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МЕТОД РОЗВ’ЯЗАННЯ БАГАТОКРИТЕРІАЛЬНОГО НЕМАРКІВСЬКОГО ЗАВДАННЯ ОПТИМІЗАЦІЇ ПОРТФЕЛЯ ПРОЄКТІВ

І. В. Кононенко, А. С. Корчакова

Предметом вивчення у статті є моделі та методи оптимізації портфеля проєктів організації для планового періоду з урахуванням післядії від раніше прийнятих рішень. Оптимізація портфелів проєктів є одним із відповідальних та складних завдань, яке вирішує вище керівництво компанії. На підставі аналізу відомих робіт у цій галузі сформульована **мета** статті: створити метод, який дозволяв би вирішувати багатокритеріальне завдання оптимізації портфеля проєктів для планового періоду з урахуванням післядії від раніше прийнятих рішень. **Завдання** статті: удосконалити метод розв'язання задачі оптимізації портфеля проєктів з точки зору максимізації різниці між доходами та витратами по всіх проєктах, що розпочинаються протягом планового періоду, запропонувати метод вирішення задачі оптимізації портфеля проєктів з точки зору соціального ефекту від проєктів, які розпочаті протягом планового періоду, створити метод вирішення задачі оптимізації портфеля проєктів для планового періоду в багатокритеріальній постановці. У статті отримано такі **результати**. Наведено математичну модель розв'язуваного завдання, цільові функції завдання включають різницю між надходженням та витрачанням коштів у часі, ризики портфеля та соціальний ефект від його здійснення. Математична модель враховує забезпечення достатності коштів у всі періоди реалізації проєктів, необхідну послідовність реалізації проєктів, обов'язкове включення деяких проєктів у портфель на

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заданому відрізку часу. Розглянута задача відноситься до багатокритеріальних немарківських динамічних задач дискретної оптимізації. Запропоновано метод її вирішення у багатокритеріальній постановці. Метод заснований на вирішенні однокритеріальних задач, а потім багатокритеріальної задачі. В основу методу покладено мінімаксний підхід та неявний перебір. **Висновки.** Розроблено метод вирішення задачі оптимізації портфеля проєктів підприємства для планового періоду за критерієм прибуток, який, на відміну від існуючих, враховує обмеження на відсутність заборгованостей та післядію від раніше прийнятих рішень. Метод послужив основою для створення методів оптимізації ризиків та соціального ефекту. Розроблено метод вирішення багатокритеріальної задачі оптимізації портфеля проєктів підприємства для планового періоду, яка, на відміну від існуючих, враховує критерії прибуток, ризики та соціальний ефект, обмеження на відсутність заборгованостей та післядію від раніше прийнятих рішень, що дозволяє підвищити якість портфеля, який формується.

Ключові слова: портфель проєктів; модель; багатокритеріальна оптимізація; плановий період; післядія; метод.

МЕТОД РЕШЕНИЯ МНОГОКРИТЕРИАЛЬНОЙ НЕМАРКОВСКОЙ ЗАДАЧИ ОПТИМИЗАЦИИ ПОРТФЕЛЯ ПРОЕКТОВ

И. В. Кононенко, А. С. Корчакова

Предметом изучения в статье являются модели и методы оптимизации портфеля проектов организации для планового периода с учетом последействий от ранее принятых решений. Оптимизация портфелей проектов является одной из ответственных и сложных задач, которую решает высшее руководство компании. На основании анализа известных работ в этой области сформулирована **цель** статьи: создать метод, который бы позволял решать многокритериальную задачу оптимизации портфеля проектов для планового периода с учетом последействий от ранее принятых решений. **Задачи** статьи: усовершенствовать метод решения задачи оптимизации портфеля проектов с точки зрения максимизации разности между доходами и затратами по всем проектам, начинаемым в течение планового периода, предложить метод решения задачи оптимизации портфеля проектов с точки зрения социального эффекта от проектов, которые начаты в течение планового периода, создать метод решения задачи оптимизации портфеля проектов для планового периода в многокритериальной постановке. В статье получены следующие **результаты**. Приведена математическая модель решаемой задачи, целевые функции задачи включают разность между поступлением и расходованием средств во времени, риски портфеля и социальный эффект от его осуществления. Математическая модель учитывает обеспечение достаточности средств во все периоды для осуществления проектов, требуемую последовательность реализации проектов, обязательное включение некоторых проектов в портфель на заданном отрезке времени. Рассматриваемая задача относится к многокритериальным немарковским динамическим задачам дискретной оптимизации. Предложен метод ее решения в многокритериальной постановке. Метод основан на решении однокритериальных задач, а затем многокритериальной задачи. В основу метода положен минимаксный подход и неявный перебор. **Выводы.** Разработан метод решения задачи оптимизации портфеля проектов предприятия для планового периода по критерию прибыль, который, в отличие от существующих, учитывает ограничения на отсутствие задолженностей и последействие от ранее принятых решений. Метод послужил основой для создания методов оптимизации рисков и социального эффекта. Разработан метод решения многокритериальной задачи оптимизации портфеля проектов предприятия для планового периода, которая в отличие от существующих учитывает критерии прибыли, риски и социальный эффект, ограничения на отсутствие задолженностей и последействие от ранее принятых решений, что позволяет повысить качество формируемого портфеля.

Ключевые слова: портфель проектов; модель; многокритериальная оптимизация; плановый период; последействие; метод.

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