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FRACTAL NATURE OF ARTERIAL BLOOD OXYGEN SATURATION DATA

The subject matter *of this study was the processing of arterial blood oxygen saturation data (SaO_2). The aim was to investigate the downsampling procedure of the SaO_2 records on a broad range of scales.* ***The object of study*** *was a small data set (20 subjects, about 164 seconds duration, sampling rate 300 Hz) borrowed from the well-known portal of medical databases Physionet.* ***The tasks to be solved are*** *a test of the dataset heterogeneity, downsampling of the SaO_2 series and its increments in a broad range of possible, checking the randomness of SaO2 series increments, argumentation in favor of applying the theory of Levy-type processes to the SaO_2 increments and proving of their self-similarity, the definition of the geometrical fractal and its Hausdorff dimension.* ***The methods used are*** *the Levy-type processes theory, statistical methods, boxes-covering method for fractal structures, the autocorrelation function, and programming within MAPLE 2020. The authors obtained the following* ***result****s: the dataset comprises three subsets with different variability; the records and their increments remain scale-invariant if the switching frequencies remain lower than the reduced sample rate; the increments of SaO_2 records are a Levy-type and self-similar random process; the fractal is the set of positions of the non-zero increments (switch-overs) from a geometrical viewpoint.* ***Conclusions****. The scientific novelty of the results obtained is as follows: 1) the fractal nature and the self-similarity of SaO_2 records and their increments were proved for the first time; 2) authors found the fractal Hausdorff dimensions for the subsets in the range (0.48…0.73) in dependence on variability; 3) authors found the principal possibility of the SaO_2 data sizes essential reducing without losses of vital information.*

Keywords: *oxygen saturation; coronavirus; downsampling; fractals; Levi processes; self-similarity.*

1. Introduction

Arterial blood gas satiety (ABG) studies most commonly are performed about oxygen (SaO_2). They have recently been paid particular attention because of the COVID-19 pandemic. Hypoxia, the shortage of oxygen, often disguised and accompanying this disease, is a reason for it [1-4]. SaO_2 trials are more difficult, expensive but direct, and exacter meanwhile [5]. The authors meant wider used peripheral oximetry (or pulse oximetry, SpO_2 [6]), making the comparison.

A typical SaO_2 time series (a blood saturation record), like most medical signals, does not comprise high-frequency components. An exception is noise, of course. Hence, a typical sampling rate also might not be too high. Thus, the downsampling is helpful in the handling of oversampled medical signals with minimal information losses.

Let here and further mean the digital, or discrete-time, signals of the computer age. What is mainly lost there at the downsampling? Firstly, a certain number of points on the time domain, depending on the downsampling factor. Secondly, the frequency resolution drops, and minimal frequency rises in the frequency domain. So, one gets the possibility to save the peculiarities of a larger scale, losing small details here and there. Besides, one can reduce the dataset size to the benefit of its storage and transfer.

Downsampling is a similarity mapping that increases the distance between setpoints from a mathematical point of view [7]. If the actual process is self-similar, then one can get a sequence of fractional copies of the process, which are more or less similar to the prototype. Of course, in a specific range of scales, that is, downsample factors.

Self-similarity or fractal nature is inherent in general metabolic processes [8]. In particular, the authors of [9] noted some fractal features of SpO_2 variability in healthy adults. Authors working with fractals disagree. They still cannot offer a unified definition of fractals. Therefore, the choice of essential features of fractals [7] is still, shall we say, "a matter of taste and convenience."

The SaO_2 dataset [10] displays heterogeneity: three subsets differing vitally by the variability of blood oxygen saturation analogous to the SpO_2 data [11]. Besides, the non-zero increments of any SaO_2 series within each of the above subsets match the Poisson distribution with three various flow intensities of events [10].

2. Aim and tasks

Now we can formulate the main aim of the article. The downsampling in its bonds with the fractal nature of the arterial blood oxygen saturation process is the focus of research. We mean here both the SaO_2 series

and its increments as the subjects of the study. A kind of "action plan" for such an aim includes the following five stages:

– test of the heterogeneity and definition of the well-grounded allocation on subsets for data [10];

– downsampling of the SaO_2 series and its increments in a wide range of possible scales with an analysis of changes;

– checking the randomness of SaO_2 series increments by autocorrelation functions;

– argumentation in favor of applying the theory of Levy random processes [12] to actual distributions of increments;

– searching of the geometrical fractal and its Hausdorff dimension.

3. Methods: data set and computing means

There we consider SaO_2 data for the small dataset (n=20 persons) from data [13] submitted in [14]. The "2018 PhysioNet/Computing in Cardiology Challenge" database was contributed in 2018 by two Massachusetts General Hospital's (MGH) laboratories.

MGH has gathered this database included 1,985 subjects monitored for sleep disorders. The data were partitioned into balanced training (n = 994), and test sets (n = 989). The reader can find more details describing this database in the sources [14, 15].

3.1. Data set selection and pre-processing

At first, 25 anonymous persons were selected randomly from the above test set. Then, the artifacts with either massive arrays of heavy outliers (3 records) or without increments (2 records) were sifted out. The dataset was cut off to 20 persons in this way. Note, the gotten dataset overlaps but is not quite identical to [10] one. There, in particular, figured one record without increments, that is, pure constant.

Besides, we focused on relatively short-time (about 3-minute duration) measure spans, though the complete records were up to one hour long. The primary sampling rate was equal to 200 Hz [14]; thus, the 3-minute duration of a signal means 36000 samples. For the comfort of downsampling, we have decreased this number up to a lower value: $N_0 = 32768 = 2^{15}$.

3.2. Computing means

All computing, including the graphics, was performed within Maple 2020 [16]. All necessary programs were also written within this computer mathematics system, albeit mostly its program packages mainly were enough for our research.

A discrete-time will be in use here and below. Let N_0 be the initial number of samples (points), and S_0^d is the primary sample rate before the downsampling. Then the discrete moments (points, samples) are:

$$t_{j,0} = \frac{j}{s_0^d}; \quad j = \left(1, 2, ..., N_0\right). \tag{1}$$

Suppose the factor of downsampling has such a form :

$$c = 2^m; \quad m = \left(0,1,...,\log_2\left(N_0\right)\right). \tag{2}$$

Let m become the title of the logarithmic factor of downsampling. Then $m = 0$ matches to the initial record, and non-zero values match to downsampled one:

$$t_{i,m} = \left(\frac{i}{s_o^d}\right) \times c; \quad i = \left(1,2,...,N_m = \frac{N_0}{c}\right). \tag{3}$$

Thus, the distance between the two nearest neighboring points (samples) increases by 2^m .at the downsampling. Their number and sample rate decrease by 2^{-m} at the same time. To be sure, the duration of observation (T) stays invariant:

$$T = t_{N_0} - t_{1,0} = t_{N_m} - t_{1,m}. \tag{4}$$

4. Results

4.1. Heterogeneity of the dataset and distribution on subsets

It would be possible to show the heterogeneity of the dataset by box plots similar to [10, 11]. The interquartile range (IQR) as the most robust statistical measure of variability is the base of such charts. Albeit, one can use and another way, based on IQR, but through the so-called normal plot. Such a plot allows comparing the factual distribution of a random value with the normal Gauss one.

Figure 1 presents the normal plot for interquartile ranges of the dataset. The horizontal axis shows the expected values, assuming the normal Gauss distribution, the vertical one - the observed values. One can see the clear separation of dots, which matches the different personal records into three subsets with divers variability. Dots form a typical "devil's stairs" with three rungs.

The variations inside each subset are minor. The one exception for the upper subset with maximal varia-

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bility might be considered an ordinary outlier. Still, the differences among the subsets are statistically significant on the 0.99 confidence levels (p-values<0.001).

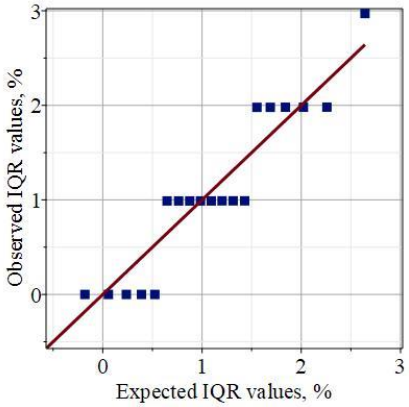

Fig. 1. The normal plot for interquartile ranges (IQR) of the dataset subjects. The thin line matches the Gauss distribution, the dots – the factual one

The bulks of subsets (6, 8 and 6 in order of ascending variabilities) are evident from Fig. 2. Since grouping tactics were different, the values are not precisely equal to those (7, 9 and 4) reported in [10]. We shall be back yet to this divergence a bit later. The results will be presented mainly for two subsets with minimal and maximal variabilities because the same results for the middle subset are intermediate.

4.2. Downsampling of SaO_2 records and their increments

Figure 2 presents typical SaO_2 records and their increments for two subjects from subsets with minimal and maximal variabilities at different logarithmic downsampling factors (see expression (2)). The upper two rows match a subject with two observed levels of oxygen saturation and 11 non-zero increments. Each non-zero increment is a switch-over between these two levels [10]. Two lower rows of Fig. 2 matrix match another subject with ten observed levels and 37 switch-overs among them.

Fig. 2 reflects the diversities between subsets with minimal and maximal variabilities of the blood oxygen saturation. Besides, one can mark that the graphs of the first two columns are well similar within their rows, whereas the graphs of the third columns of each row look like fairly "distorted" copies from the first and second columns.

This distortion effect is especially evident for increments. Their number on the interval of observation has to be invariant. Besides, the interval itself also is constant at downsampling (see expression (4)). Hence, the frequency of switch-overs, that is, the frequency of Poisson's flow of events [10], is saved too.

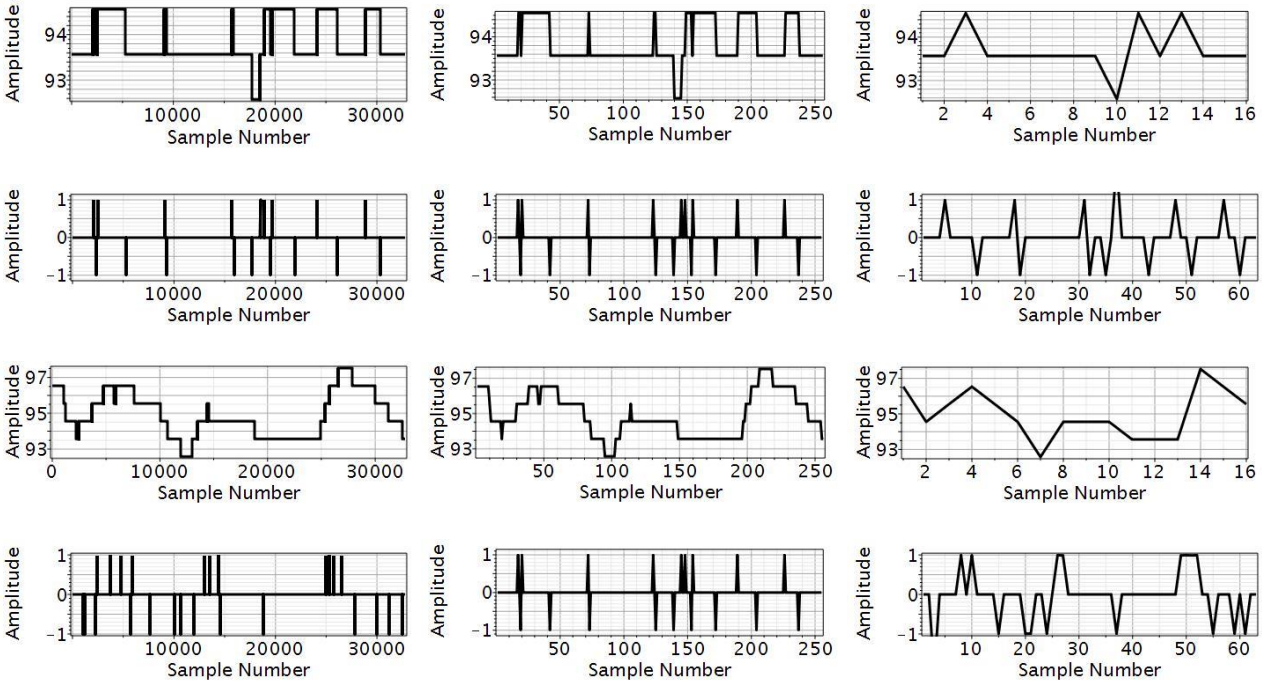

Fig. 2. SaO_2 records (the first and the third rows) and their increments (second and fourth rows) for two subjects of the subsets with minimal variability (first and second rows) and with the maximal one (third and fourth rows). The columns match the different logarithmic factors of downsampling: 0, 7 and 9 from the left to the right respectively. All amplitudes are given in percent

The maximal number of switchings is observed in the subset with maximal variabilities and is equal to 42. It matches the switching frequency of 0.256 Hz. Thus, the reduced sample rate has to be at least twice higher than this value. One can get, taking into mind that $s_0^d = 200$ and (2) the following inequality:

$$0.256 \le \frac{200}{2^{m+1}}. \qquad (5)$$

This inequality has positive integer-type solutions only from the range $m = [0,1,...,9]$. So, the maximal logarithmic factor of downsampling is limited and equal to 8 in our case. One can ensure the similarity of downsampled signals to the original one and the invariance of switching frequencies only up to this factor.

What if it will be exceeding? Then the known effect of aliasing will begin, which the reader can see in the third column of Fig. 2. This effect distorts signals dropping down even the numbers of switching, which have to be an invariant of downsampling.

The reader can convince of the random nature of switch-ower processes by the analysis of the autocorrelation functions of increments (Fig. 3).

Note that downsampling is very helpful in the case of computing correlograms, recurrence plots, data matrices, etcetera. The point is that such calculations need the machine-time that is rising as $O(n^2)$. It being "over and above" if the series is lengthy enough. In particular, if we were to calculate these correlograms for the primary data (m = 0, n = 32768), this would take away much time with an ordinary computer.

4.3. Does the switch-over process be one of the Levy random ones?

A random process ($X = \{X_t : t \ge 0\}$) is the Levy one if [12]:

i. it has the right limit ($X_{t+} = X_t$) and existing left limit (X_{t-});

ii. it is initiated from the origin ($X_0 = 0$);

iii. for $0 \le s \le t$, $X_t - X_s$ is equal in distribution to X_{t-s};

iv. for $0 \le s \le t$, $X_t - X_s$ is independent of $\{X_u : u \le s\}$.

The switching processes, presented by Fig. 2 (the second and fourth rows) as increments of blood saturation, are continuous in probability (i). Although the spectra of possibilities are discrete and even binary, they can be equal to zero or non-zero. The process begins from the origin (ii), has stationary (iii), independent (iv) increments. Yes, it is a Levy random process in such a case and sense.

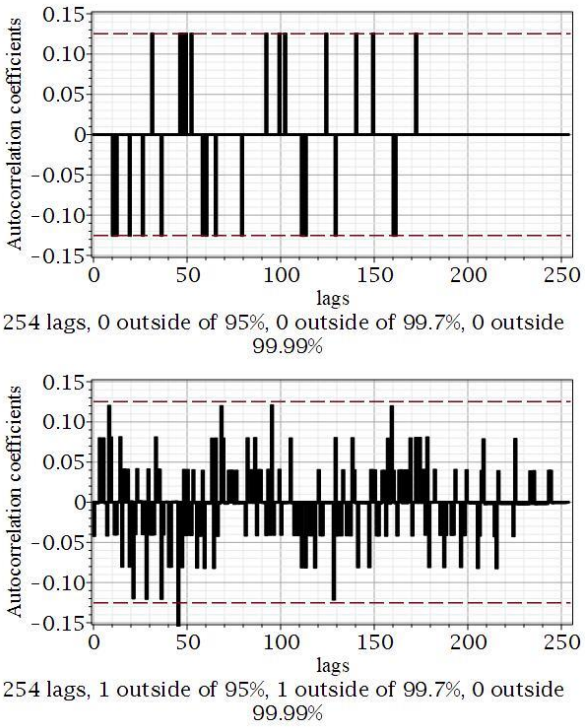

Fig. 3. Normalized correlograms for switching processes: the upper plot relates to the subject from the minimal variability subset, the lower one to the subject from the opposite subset. The dash lines note the 95 % confidence intervals

Two mainly discussed patterns of Lévy processes are the Wiener process, else often called the Brownian motion one, and the Poisson process. The divergence is in the probability distributions: either Gauss' or Poisson's. Note that this difference is minor for the large enough numbers of trials. In this case, the Poisson distribution tends to Gauss one. That is the statement of the central limit theorem (CLT) of the probability theory.

What should it mean in this context, "a large enough number of trials?". The answer is inequality (5). Let rewrite it in terms of switching numbers (n) versus numbers of points on the observing interval after the downsampling (N_m). Then one will get:

$$N_m > n. \qquad (6)$$

The lower the downsampling factor, the more potent the last inequality, and any Poisson process will be closer to the Wiener one.

Neither Gauss nor the Poisson distribution describes the probabilities of switch-overs (increments),

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precisely at least. Both are unimodal, whereas the histograms and estimations of probability density function for increments point out three existing modes: primary mode for zero probability and an additional two lower for positive and negative non-zero probabilities.

Neither Gauss nor the Poisson distribution describes switching probabilities (increments), at least not precisely. Both are unimodal, while histograms and probability density function estimates for the increments indicate three existing modes: the primary mode for zero probability and two additional lower ones for positive and negative non-zero probabilities.

However, these distributions can still be an excellent approximation to reality. Lets show that the switching process, which is already clear, is of the Levy type, is quite close to the same Wiener type. Firstly, any Wiener process has zero expectation:

$$E[W_t] = 0. \quad (7)$$

Any switching process also has a zero-equal expectation. It is evident from Fig. 2 and simply testable. Hence, the variances of both processes are just the expectation of their quadrates. Secondly, any Wiener process is self-similar, thence:

$$E[W_t^2] = \left(\frac{1}{c}\right) E[W_{ct}^2], \quad (8)$$

where c is the scale coefficient. It means that the variance of the Wiener process is c-time lesser than for the scaled Wiener one.

Taking into mind (2), one can get the following dependence of the variance on logarithmic downsampling factor for a self-similar process with zero expectation:

$$Var(X(m)) = 2^m \times Var(X(0)). \quad (9)$$

Fulfilling condition (9) for the switch-over process would mean that this process is self-similar, like the Wiener one. Fig. 4 shows that such an assumption has a solid fundament.

The expression (9) looks like a straight line with a slope equal to unity within the semi-logarithmic coordinates if one uses the logarithms with a base equal to 2. The slope will be equal to $\ln(2) \approx 0.693$, if one uses the natural logarithms. Pay attention to the dependencies in Fig. 4. They are straight lines, but only in a limited range, introduced above. Table 1 testify that the slopes of all three lines are virtually equal to the unity.

Thus, the switching process for blood oxygen saturation levels is Levy-type and self-similar in the defined range of downsampling factors (scales). Nevertheless, it does not equal neither the Wiener process nor the Poisson one by distribution. Here we mean increments of SaO_2, speaking about the random process. That is why the other values, switching number, for instance, can have almost the Poisson distribution within subsets [10].

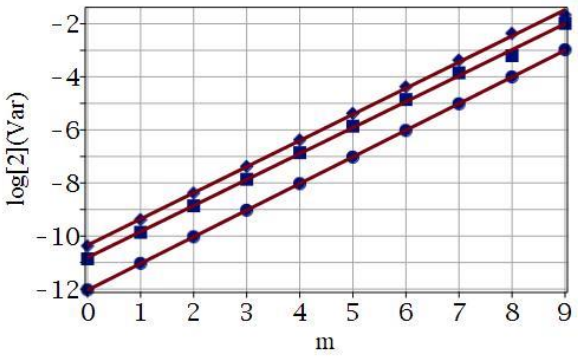

Fig. 4. The dependences of variations in subsets on the logarithmic factor of downsampling in semi-logarithmic coordinates: the lower line matching the subset with minimal variability, the upper one - the subset with maximal variability

Table1

The slopes of the lines of Figure 4

Subsets	Slopes	Standard deviation
Minimal variability	1.004	0.002
Middle variability	0.980	0,010
Maximal. variability	0.986	0.009

4.4. Where are the genuine fractals here?

The dependences of Fig. 2 are multi-fractals, more precisely bi-fractals, since functions of blood oxygen saturation have two types of singularities: "up" and "down"[10, 17, 18]. Functions of Fig. 2 have zero-equal derivatives almost everywhere. "Almost" in this case means with except a set of positions (points) with zero Lebesgue measure [10, 17].

These sets of positions are one-dimensional fractals by themselves [10, 17]. Hence, each of them has a Hausdorff dimension between 0 and 1. One can find the fractal dimension by the known covers method [19] in one dimension space. The number $N(r)$ of non-empty segments of length (scale) r needed to cover the fractal set of points depends on r as:

$$N(r) \sim r^{-D}. \quad (10)$$

The relation (10) is the scaling law, and D is the fractal (Hausdorff) dimension. Table 2 shows the gotten Hausdorff dimensions.

We have obtained the Hausdorff dimension of Table 1 under the condition m = 0. Although the fractal will remain invariant in the range m = [0,1, ... ,9]. Further, the positions within the fractal begin to merge and blur (see Fig.2). One can evaluate the quality of the scaling law (10) fulfillment by determination coefficients (R^2): as close is it to 1 as the law fulfills better. The range of scales was $r \in [5,...,100]$ seconds in our case.

Table 2

Fractal (Hausdorff) dimensions of the positions where the switch-over of oxygen saturation levels take places

Subsets	D	Standard deviation	R^2
Minimal variability	0.48	0.03	0.97
Middle variability	0.70	0.08	0.92
Maximal variability	0.73	0.06	0.95

All estimations are between 0 and 1. That is, switchovers fractal dimension is between points and lines. The range of values for D (Table 1) is close to the fractal dimensions of random "Cantor dust" [20].

Note, these estimations are not too precise. The reason is a small number of switch-overs on the watched interval (from 8 to 38). Nonetheless, the subset with minimal variability has the lower fractal dimension. Its dimension is closer to the dimension of a Wiener process zeros (0.5 [20])

4.5. Discussion of results

Fractality and self-similarity of fractal structures are crucial to the insight of the above results [10, 19, 21]. Many physiological systems such as blood vessels, breathing airways and vessels, pulmonary alveoli system, molecules of deoxyribonucleic acid (DNA), heartbeats, and even gait patterns all show fractal characteristics as well [21]. However, the fractal nature of blood oxygen saturation had begun to clarify recently [9 – 11].

We used the most reliable statistical measure of variability to distribute subjects across subsets, i.e., interquartile differences. This way differs from the method [10], where the selection was by the closeness of the numbers of switchings. Therefore the capacities and composition of the subsets turned out to be somewhat different also. For example, we got (6, 8 and 6) subjects in the three subsets against (7, 9 and 4) in [10].

Here it is appropriate to discuss the capacity of the dataset used in the article (20 persons). It is relatively small. Still, it turned out quite enough to detect its heterogeneity, classify switching process type, and prove its self-similarity. In other words, the capacity of this dataset is enough for qualitative studies. On the other hand, such a capacity of the dataset is not enough for quantifications, discussed in the previous paragraph. It touches on our first estimations of Hausdorff dimensions: they are not too precise. Such estimates need an extended dataset.

The step-like shape of signals, discrete and limited spectra of observed saturation levels (from 2 to 10), the same amplitudes of increments are inherent in the SaO_2 series. The increments are independent and identically distributed random samples. A non-zero increment is a rare event in comparison with the number of trials (N_m).

Positions of the blood saturation levels switchovers are forming the one-dimensional stochastic fractal. This fractal preserves its structure in a wide enough scale of downsampling factors. As a result, such values as the number of switchings, observed saturation levels, relations among the inter-switch time intervals, and switching frequencies turned out scale-invariant.

The switching of the oxygen saturation levels has all hallmarks of the random self-similar process of Levy-type. Moreover, the right continuous probability ensures this process is also strong Markov-type [12, 22]. Still, this process is neither Wiener nor Poisson exactly. Though the Poisson distribution roughly depicts the number of switchings on the interval [10].

This fractal has a Hausdorff dimension differing from zero (its Lebesgue dimension). Curiously the fractal dimension of the subset with minimal variability is almost equal to zero of a Wiener process (0.48 against 0.5 [20]). The dimensions of the two subsets are higher (0.7…0.73), and their difference with the previous subset is statistically significant.

Thus, downsampling of SaO_2 data over a wide range might be without losing essential information about the oxygen saturation process. Only the known Nyquist–Shannon condition for frequencies limits the upper bound of the downsampling ratio. More compact data is easier to transfer and process. It might be helpful in clinical practice, especially at the pandemic time.

The proving of self-similarity of SaO_2 increments is one of the new results in the problem. Besides, the authors, the first time, detected the Levy-type and Markov-type of these processes. They pointed out the one-dimensional geometrical fractals (the switching positions) and also made estimations of their Hausdorff dimensions.

5. Conclusions

Let us formulate the conclusions according to the tasks from the introduction section.

1. The randomized SaO_2 data set, purified from artifacts, has natural heterogeneity. The data set has three subsets varied by their variability, which is statistically

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significant. The relative fractions of subsets are 30 %, 40 %, and 30 % in variability ascending order.

2. Downsampling saves the SaO_2 series and their increments and many of their parameters. They turned out scale-invariant in the wide range of downsampling factors (scales). The well-known Nyquist-Shannon condition defines the upper bound of the downsampling scales.

3. The increments of the SaO_2 series, describing the switch-overs among the observed saturation levels, are random, independent, and identically distributed. They have identical magnitudes in addition.

4. The switching process is a random Levy-type one, which is self-similar. However, it does not precisely match Wiener or Poisson processes because it has three-modal distribution.

5. The set of positions (points, time moments) for non-zero increments (switching) is a stochastic, one-dimensional fractal. Its Hausdorff dimensions are between 0 and 1 and vary for three subsets. The fractal preserves itself at the downsampling.

Future research directions

Nowadays, the authors are seeing two straight paths concerning the onward perspectives of the research. It is the study of pathological blood oxygen saturation, especially linked with COVID-caused pneumonia. Besides, it is a more broad study of self-similarity probably inherent in many medical signals.

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ФРАКТАЛЬНА ПРИРОДА ДАНИХ НАСИЧЕННЯ АРТЕРІАЛЬНОЇ КРОВІ КИСНЕМ

Г. П. Чуйко, Є. С. Дарнапук

Об'єктом статті є обробка даних про насичення артеріальної крові киснем (SaO_2). **Мета** полягає в тому, щоб дослідити процедуру зниження частоти дискретизації записів SaO_2 в широкому діапазоні масштабів. **Предметом дослідження** був невеликий набір даних (20 осіб, тривалість близько 164 секунди, частота дискретизації 200 Гц), запозичений з відомого порталу медичних баз даних Physionet. Вирішуються завдання: перевірка неоднорідності набору даних, зниження дискретизації ряду SaO_2 та його приростів у широкому діапазоні можливих, перевірка випадковості приростів ряду SaO_2, аргументація на користь застосування теорії процесів типу Леві. до приростів SaO_2 та доведення їх самоподібності, визначення геометричного фракталу та його хаусдорфової розмірності. **Використані методи**: теорія процесів типу Леві, статистичні методи, метод покриття блоків для фрактальних структур, автокореляційна функція, програмування в рамках MAPLE 2020. Автори отримали такі **результати**: набір даних складається з трьох підмножин з різними варіабельностями; записи та їх прирости залишаються незмінними в масштабі, якщо частоти перемикання залишаються нижчими, ніж зменшена частота дискретизації; збільшення записів SaO_2 є випадковим процесом типу Левіб до тогож самоподібним; фрактал — це набір положень ненульових приростів (перемикань) з геометричної точки зору. **Наукова новизна отриманих результатів** полягає в наступному: 1) вперше доведено фрактальну природу та самоподібність записів SaO_2 та їх приростів; 2) автори знайшли фрактальні розмірності Хаусдорфа для підмножин в діапазоні (0,48…0,73) в залежності від вапріабельності; 3) автори виявили принципову можливість істотного зменшення розмірів даних SaO_2 без втрат життєво важливої інформації.

Ключові слова: насичення киснем; коронавірус; зниження дискретизації; фрактали; процеси Леві; самоподібність.

ФРАКТАЛЬНАЯ ПРИРОДА ДАННЫХ НАСЫЩЕНИЯ АРТЕРИАЛЬНОЙ КРОВИ КИСЛОРОДОМ

Г. П. Чуйко, Е. С. Дарнапук

Объектом статьи является обработка данных о насыщении артериальной крови кислородом (SaO_2). **Цель** состоит в том, чтобы исследовать процедуру понижающей дискретизации записей SaO_2 в широком диапазоне масштабов. **Предметом исследования** был небольшой набор данных (20 человек, длительность около 164 секунд, частота дискретизации 200 Гц), заимствованный из известного портала медицинских баз данных Physionet. **Решаемые задачи**: проверка неоднородности набора данных, даунсэмплинг ряда SaO_2 и его приращений в широком диапазоне возможных, проверка случайности приращений ряда SaO_2, аргументация в пользу применения теории процессов типа Леви. к приращениям SaO_2 и доказательство их самоподобия, определение геометрического фрактала и его хаусдорфовой размерности. **Используемые методы**: теория процессов типа Леви, статистические методы, метод покрытия прямоугольников для фрактальных структур, автокорреляционная функция, программирование в рамках MAPLE 2020. Авторами получены следующие **результаты**: набор данных состоит из трех подмножеств с различной изменчивостью; записи и их приращения остаются масштабно-инвариантными, если частоты переключения остаются ниже, чем уменьшенная частота дискретизации; приращения записей SaO_2 представляют собой самоподобный случайный процесс типа Леви; фрактал - это набор положений ненулевых приращений (переключений) с геометрической точки зрения. **Научная новизна полученных результатов** заключается в следующем: 1) впервые доказана фрактальная природа и самоподобие записей SaO_2 и их приращений; 2) авторы нашли фрактальные размерности Хаусдорфа для подмножеств в диапазоне (0,48…0,73) в зависимости от изменчивости; 3) авторы обнаружили принципиальную возможность существенного уменьшения размеров данных SaO_2 без потерь жизненно важной информации.

Ключевые слова: насыщение кислородом; коронавирус; даунсэмплинг; фракталы; процессы Леви; самоподобие.

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