UDC 519.6: 629.7.036.3

A.R. YAMATOV[1], S.A. PLESOVSKIH[1], S.F. TYURIN[2]

[1]***Perm Military Institute of the Interior Ministry of Russia, Russia***
[2]***Perm national research polytechnic university, Perm, Russia***

THE TECHNIQUE OF BUILDING STRUCTURAL SCHEMES OF SYSTEM RELIABILITY USING WITH MODIFIED GRADIENT FOR THE PROCEDURE OF THE STEEPEST DESCENT

New technology using a procedure with a modified steepest descent gradient to build the block diagram of the system reliability. There are a comparative analysis of the data obtained by other methods. The were investigation of the effectiveness of methods with the use of the steepest descent procedures and dynamic programming in the designing of the block diagram of system reliability. Identified additional stages to the method of steepest descent improving quality reliability block diagrams (RBD), and in the most cases lead to an optimum RBD. The application of the method and procedure of adaptation of the gradient provides essential lowering informational complexity of the algorithm for finding the optimal solutions.

Key words*: the steepest descent, modified gradient, dynamic programming, design of the block diagram of reliability, optimization, redundancy.*

The introduction

The problem of ensuring the required reliability of the device is associated with all phases of his life: design, development and practical use. In the design phase providing of the required reliability is achieved by methods that do not require reservations [1].

In cases where such methods of increasing reliability of the device have been exhausted, but is not available from the specified parameters, such as the set time between failures, in order to further improve the reliability of resorting to reservations.

An actual problem when designing of optimal control systems and reliability block diagrams (RBD) is to provide highly reliability with limited resources. By resources in this case understand the cost, occupied volume and weight of the system, etc.

Therefore, actual scientific problem is the development of new effective mathematical methods and algorithms for constructing an optimal system for reliability by criterion when the given reliability is achieved at the lowest possible amount or value of the minimum reserve equipment, or for a given volume or value of redundant equipment will be reached the highest possible reliability.

The GOST [2] establishes general rules for calculating the reliability of technical objects, methods and requirements for the presentation of the results of calculation of reliability, but does not define methods of design for RBD system.

The task of designing the optimal RBD can be solved by the Bellman's method (dynamic programming) [3, 4], the method of steepest descent [5, 6, 7] and by using a genetic algorithm [8]. Often such solutions are close to optimal parameters due to the peculiarities of their use.

Of the above methods were chosen Bellman's method (dynamic programming) and the procedure of steepest descent, as both methods are relatively easy to manual calculation and implementation as a program. The purpose of this article is to prove the advantages of the steepest descent method with a modified gradient and to identify ways to further optimize RBD system.

Redundant system is called optimal for reliability criterion if the specified reliability is achieved at the lowest possible amount or value of the minimum reserve equipment, or for a given volume or value of redundant equipment will be reached the highest possible reliability.

1. The definition of the gradient for realization of the steepest descent

Simplified method designing RBD system based steepest descent procedure is described in [6,7]. In the methodology used the gradient:

$$(\delta_i^j)^* = \max\{\delta_i^j\} \text{ for } i = \overline{1,5},$$

$$\delta_i^j = \frac{P_i^{j+1}(t) - P_i^j(t)}{W_i \cdot P_i^{j+1}(t)}, \tag{1}$$

where j – iteration number, starting with 0 – this RBD obtained in the first stage; P_i – state probability (SP) subsystem; W_i – the cost of subsystem.

Gradient (1) does not include the use of majority reservation and the methodology are not considered methods to further improve the results. In the chapter 8.8.4 [4] to calculate the gradient, if the variables have different units, it is proposed to move to the relative variables y_i, using the minimum and maximum possible values of variables x_i:

$$y_i = \frac{x_i - x_i^{min}}{x_i^{max} - x_i^{min}}, \quad (2)$$

Such use of the gradient in the form of a «growth factor cost» was found in the method of optimizing network models in terms of «Time - the cost of» [9].

$$k(i,j) = \frac{C_п(i,j) - C_н(i,j)}{T_н(i,j) - T_y(i,j)}, \quad (3)$$

where k (i, j) - rate of increase of costs showing cost of funds necessary to reduce the duration of the work (i, j) on one day; $C_п(i, j)$ - $C_н(i, j)$ - the difference between the an elevated and the «normal» cost of the work; $T_н(i, j)$ - $T_y(i, j)$ - the difference between «normal» and the accelerated time performance.

Proposed a modified gradient RBD system allows to obtain with similar optimal parameters and methodology for the further optimization of RBD:

$$(\delta_i^j)^* = \max\{\delta_i^j\} \text{ for } i = \overline{1,5},$$

$$\delta_i^j = \frac{P_i^{j+1}(t) - P_i^j(t)}{W_{i+1} - W_i} \quad (4).$$

2. The method of designing the RBD for using a modified gradient and the stages it optimizing

Let the system includes in the structure **n** subsystems. Known values of SP P_i and cost W_i (i = 1, ..., n) of each subsystem. Model of the problem will be in the form:

$$P_c(t) = \prod_{j=1}^{N} \varphi_j(m_j) \to \max, \quad (6)$$

where $P_c(t)$ - SP of the desired system;

$\varphi_j(m_j)$ - SP j-th block with m_j duplicate elements.

$$W_c = \sum_{j=1}^{N} W_j m_j \le Q, \ \forall m_j \ge 0, \text{ int}, \quad (7)$$

where W_c – the cost of desired system.

There are two formulations of the optimization problem of RBD system:

1) To build a redundant system elements by

$$W_c \to \min \text{ with } P_c(t) \ge P_c^z(t), \quad (8)$$

where $P_c^z(t)$ – given SP system.

2) To build a redundant system elements by

$$P_c(t) \to \max \text{ with } W_c \le W_c^z, \quad (9)$$

where W_c^z – given cost of system.

In the first stage of optimization for the first criterion enforce the terms $P_i(t) \ge P_c^Z(t)$, that is SP each subsystem should not be worse than a given.

In the second stage iteratively increase reserves for the largest increment SP on unit $(\delta_i^j)^* = \max\{\delta_i^j\}$.

If the condition $P_c(t) \ge P_c^Z(t)$ is not satisfied, repeat step 2.

Next, we find the cost $W_ñ^{min}$ of implementing the system when achieved $P_c(t) \ge P_c^Z(t)$.

In applying the technique to a modified gradient (4) for the calculation of the systems with limited value and majority redundancy (MR) of the first element, RBD optimal in 44% of cases (with a gradient of (1) 36%). Described below methods further optimization are for the 95% match with the best RBD.

In applying the first element of MR, with some initial data backup solution leads to 3 from 5, although reserve 2 of 3 already had enough [10]. In such cases, it is proposed to repeat the calculations from step where MR 2 out of 3 is applied by ignoring δ_1^j.

This technique has also been adapted for the designing of RBD limited value, and the results of the experiment in the most cases optimal or similar to them (the deviation value is not more than 8%).

Supplement the method stages increase the quality of RBD.

In the third stage, if the result is close to the optimum, and for one or more blocks of satisfies the condition

$$W_c^Z - W_c \ge W_i, \quad (10)$$

then select one unit with a minimum reached probability $\varphi_j(m_j)$ for further redundancy.

No more than 5% of the method leads to extremely close decision (RBD), and the condition $W_c^Z - W_c \ge W_i$ is not satisfied. These solutions are found in the application of MR first element.

In the **fourth stage**, we solve the problem to eliminate such «non-optimal» solutions. In this case proposed recalculation using a modified gradient (4) reduced to the form:

$$\delta_i^j = \frac{P_i^{j+1}(t) - P_i^j(t)}{W_i}, \quad (11)$$

ie as for power redundancy replacement. After that, under the condition (10) holds the third stage. The third and fourth stages in 95% of cases result in optimal RBD, which coincides with the solution found «brute force», and the remaining 5% of the received RBD extremely close to optimal.

The use of the proposed method will reduce the computational complexity of the designing of RBD to a few iterations, and get a system with the optimal values of the desired parameters.

3. The example of using method

We solve the problem for the criterion $P_c(t)$, in order of definition the optimal strategy of duplication within the specified limits.

Let the automation system includes in its membership five subsystems, under certain values SP P_i and cost W_i, where $i = \overline{1,5}$ for each device.

In designing optimal RBD allowed: majority redundancy of the first subsystem at the initial stage of optimization and redundancy replacement with a loaded mode of operation of other elements on the other steps. If necessary, can be replaced a MR on redundancy replacement with a loaded operating mode elements at an early stage, or the use of MR 3 of 5; redundancy replacement with a loaded mode of operation for the subsystems elements 2,3,4, redundancy replacement with a loaded or unloaded operation mode redundant elements for subsystem 5. Unreliability and cost of majority elements and switching devices can be neglected.

The setpoints SP subsystems: **P_1=0,9, P_2=0,75, P_3=0,82, P_4=0,8, P_5=0,9;** their value **W_1=16, W_2=11, W_3=13, W_4=12, W_5=15** respectively.

Given value of SP system **$P_c^z(t)$ = 0,94;** set (operating) cost value system **W_c^z = 120**.

The solution

Find $W_c \to \min$ for $P_c(t) \geq P_c^z(t)$.

For the beginning verify that of the $P_i(t) \geq P_c^z(t)$ conditions for each **i** from 1 to 5.

It can be seen that none of the sections of this condition is not satisfied, so it is necessary the introduction redundant elements.

In the first stage we get the following optimization RBD – 3,2,2,2 (reference system), where

$$P_1(t) = 3P_1^2 - 2P_1^3 = 0{,}972 > P_c^z(t).$$

$$P_2(t) = 1-(1-P_2)^3 = 0{,}9844 > P_c^z(t).$$

$$P_3(t) = 1-(1-P_3)^2 = 0{,}9676 > P_c^z(t).$$

$$P_4(t) = 1-(1-P_4)^2 = 0{,}96 > P_c^z(t).$$

$$P_5(t) = 1-(1-P_5)^2 = 0{,}99 > P_c^z(t).$$

Find SP system and its value to 0–step.

$$P_c^0 = 0{,}972 \cdot 0{,}9844 \cdot 0{,}9676 \cdot 0{,}96 \cdot 0{,}99 = 0{,}8799$$

$$W_c^0 = 3 \cdot 16 + 3 \cdot 11 + 2 \cdot (13+12+15) = 161.$$

Now that each subsystem has a SP larger or equal set, go to the second stage – we need to increase the SP of the system. Try to increase the SP for one subsystem. As permitted to use non-adaptive majoritarian redundancy now have to enter the 5 channels and choices 3 of 5:

$$P_1^1(t) = P^5 + 5P^4(1-P) + 10P^3(1-P)^2$$

On other sections – reservation replacement with a loaded operating mode redundant subsystems, ie $P_i^1(t) = 1-(1-P_i)^n$ – we have, at the least, one of the available channel. We get:

$$P_1^1(t) = P^5 + 5P^4(1-P) + 10P^3(1-P)^2 = 0{,}99144$$

$$P_2(t) = 1-(1-P_2)^4 = 0{,}996$$

$$P_3(t) = 1-(1-P_3)^3 = 0{,}994$$

$$P_4(t) = 1-(1-P_4)^3 = 0{,}992$$

$$P_5(t) = 1-(1-P_5)^3 = 0{,}999$$

Using a modified gradient (2) is defined $(\delta_i^j)^* = \max\{\delta_i^j\}$. $(\delta_i^1)^* = \delta_4^1$, then the next element should be added fourth section. Therefore, RBD for step j = 1 will have the form 3,3,2,3,2.

$$P_c^1 = 0{,}972 \cdot 0{,}9844 \cdot 0{,}9676 \cdot 0{,}992 \cdot 0{,}99 = 0{,}909 < P_c^z(t)$$

$$W_c^1 = W_c^0 + W_4 = 173.$$

Thus, increasing the reserve only in the fourth section, and the rest are unchanged.

Step by step, reserving items listed modified gradient (2), we get RBD 3,4,3,3,4, for which

$$P_c^3 = 0{,}972 \cdot 0{,}9961 \cdot 0{,}994 \cdot 0{,}992 \cdot 0{,}99 = 0{,}945 \geq P_c^z(t),$$

that satisfies $P_c(t) \geq P_c^z(t)$.

The cost of implementing the system on the third step of optimization $W_c^3 = W_c^2 + W_2 = 197$.

Thus, the minimum cost of implementing the system at W_c^{min} =197 achieved $P_c(t)$ = 0,945 greater than the specified 0,94.

The solution to this task with using a gradient has led to RBD 5,3,3,3,2 with $P_c(t)$ = 0,953 at W_c^{min} = 218.

The results of the frequency and value of such abnormal are described above.

Following are the results of an experimental solution to the problem with the method of steepest descent gradient (1), (4) and Bellman's method.

4. The results of the experiment on the application procedure of steepest descent and Bellman's method

Algorithms that implement the methods «brute force», steepest descent and Bellman's method were implemented in a software product.

Calculations based on a sample of 9 variants of initial data are presented below.

The results of the calculation in the application methods without MR are presented in table 1. The results of the calculation method with the application of MR of the first block are shown in table 2. Results with abnormals from those found «brute force» are in italics. Strings with the decisions presented in the order specified in the header of tables 1 and 2.

When you try to apply the MR first subsystem with limited value, the steepest descent procedure can not build a system because of lack of resources. In this case, carry out the construction of the system, not using MR.

Analysis of the results gives an indication of sufficient accuracy of the method the steepest descent with using a modified gradient. Even if you pay attention to deviations in tasks 4 and 7 (Table 1), we will see an increase SP at minimal cost.

In applying the MR quality RBD greater with a gradient (4). And the execution of 3 and 4 stages of optimization results in 95% indicators of RBD system to optimal settings.

The conclusions

Held analysis of the effectiveness of different methods for optimizing the design of the block diagram of the system reliability

Solutions found by Bellman's method by $W_c \rightarrow min$ in 80% have a deviation from the optimal 10%, and by $W_c \rightarrow min$ RBD found deviations from optimal, sometimes reaching 47%.

Procedure with a modified the steepest descent gradient (4) and further optimization allows to solve the problem of designing RBD in polynomial time with sufficient accuracy.

In the procedure of steepest descent for tasks without MR, the use of gradients (1), (4), (10) leads to identical results.

This method can be used for practical calculations, and in the educational purposes.

Transforming gradient method can be used to calculate the various systems using different criteria (weight or volume of equipment, etc.).

Table 1

The results of using method without a MR

Specified SP and cost systems		Method «brute force» Steepest descent gradient (1) (4) (10) Bellman's method					
		$W_c \rightarrow min$ with $P_c(t) \geq P_c^z(t)$			$P_c(t) \rightarrow max$ with $W_c \leq W_c^z$		
		RBD	$P_c(t)$	W_c	RBD	$P_c(t)$	W_c
1	0,95 110	32222	0,9549	120	22222	0,9241	110
		32222	0,9549	120	22222	0,9241	110
		32322	*0,9735*	*130*	*31222*	*0,9094*	*108*
2	0,93 100	22322	0,9323	120	22212	0,7846	96
		22322	0,9323	120	22212	0,7846	96
		22332	*0,9506*	*135*	21222	0,7846	97
3	0,94 120	23322	0,9448	109	23323	0,9633	119
		23322	0,9448	109	23323	0,9633	119
		23323	*0,9633*	*119*	23323	0,9633	119
4	0,92 100	32224	0,9252	136	22212	0,7661	96
		33222	*0,9394*	*139*	22212	0,7661	96
		33223	*0,9394*	*139*	*31113*	*0,6125*	*91*
5	0,94 115	32332	0,9514	170	31221	0,7406	114
		32332	0,9514	170	31221	0,7406	114
		42332	*0,9628*	*181*	41112	0,5822	115
6	0,93 105	32233	0,9349	101	32233	0,9349	101
		32233	0,9349	101	32233	0,9349	101
		32333	*0,9532*	*110*	*31333*	*0,8665*	*100*
7	0,92 110	43222	0,9267	103	33223	0,9442	105
		33223	*0,9442*	*105*	33223	0,9442	105
		33223	*0,9442*	*105*	33223	0,9442	105
8	0,93 60	32324	0,9393	82	22214	0,7915	60
		32324	0,9393	82	22214	0,7915	60
		32324	0,9393	82	*31214*	*0,7072*	*58*
9	0,94 105	42233	0,9506	123	32222	0,8848	100
		42233	0,9506	123	32222	0,8848	100
		42233	0,9506	123	*41133*	*0,7515*	*99*

Table 2

The results of using method with a MR

Specified SP and cost systems		Method «brute force» Steepest descent gradient (4) Steepest descent gradient (1)					
		$W_c \rightarrow \min$ with $P_c(t) \geq P_c^z(t)$			$P_c(t) \rightarrow \max$ with $W_c \leq W_c^z$		
		RBD	$P_c(t)$	W_c	RBD	$P_c(t)$	W_c
1	0,95 110	Нет решений			31222	0,8214	108
					31222	0,8214	108
					31222	0,8214	108
2	0,93 100	33322	0,9333	142	22212	0,7846	96
		33322	0,9333	142	22212	0,7846	96
		33322	0,9333	142	*22211*	*0,7264*	*87*
3	0,94 120	33323	0,9458	129	23323	0,9633	119
		33323	0,9458	129	23323	0,9633	119
		53322	*0,9462*	*139*	23323	0,9633	119
4	0,92 100	53324	0,9204	179	22212	0,7661	96
		53324	0,9204	179	22212	0,7661	96
		53324	0,9204	179	22212	0,7661	96
5	0,94 115	No solutions			31221	0,6348	114
					31221	0,6348	114
					31221	0,6348	114
6	0,93 105	53344	0,9321	149	32233	0,8444	101
		53344	0,9321	149	32233	0,8444	101
		53344	0,9321	149	*32232*	*0,8122*	*94*
7	0,92 110	55334	0,9202	160	53222	0,8597	109
		54434	0,9217	162	53222	0,8597	109
		54434	0,9217	162	53222	0,8597	109
8	0,93 60	53424	0,9322	106	22214	0,7915	60
		53424	0,9322	106	22214	0,7915	60
		53424	0,9322	106	*22213*	*0,7764*	*57*
9	0,94 105	No solutions			32222	0,7584	100
					32222	0,7584	100
					52122	*0,7326*	*98*

The further work can be focus on the analysis of error in the initial data and the inclusion of the gradient of the estimated variances (errors) of these results.

Transforming the gradient method can be used to calculate the various systems using different criteria (weight or volume of equipment, etc.).

Also possible to consider the resulting RBD system as a support for the solution of genetic algorithm.

References

1. Золотова, Т.М. Резервирование аналоговых устройств автоматики [Текст] / Т.М. Золотова, Ф.И. Кербников, М.А. Розенблат. – 2-е изд. – М.: Энергоатомиздат, 1986. – 128 с.

2. ГОСТ 27.301-95 Надежность в технике. Расчет надежности. Основные положения [Текст]. – Взамен ГОСТ 27.410-87 (в части п.2); введ. 01.01.1997. – М.: Изд-во стандартов, 1996. – 9 с.

3. Надежность и эффективность в технике [Текст]: Справочник в 10 т. / Ред. совет: В. С. Авдуевский (пред.) и др. Т. 2. Математические методы в теории надежности и эффективности / Под ред. Б. В. Гнеденко. – М.: Машиностроение, 1987. – 280 с.

4. Гольдштейн, А.Л. Теория принятия решений. Задачи и методы исследования операций и принятия решений: Учебное пособие. – Пермь: ПГТУ, 2002. – 357 с.

5. Luenberger, D.G. Linear and Nonlinear Programming [Текст] / D.G. Luenberger, Ye Yinyu. – 3rd ed. – Springer, 2008. –550 p.

6. Основи надійності цифрових систем: Підручник [Текст]/ за ред. В.С. Харченко, В.Я. Жихарева. –Харків: Міністерство освіти та науки України, 2004. – 572 с.

7. Надежность, контроль и диагностика ЭВМ [Текст]: Методическое пособие / В.С. Харченко, И.В. Лысенко. – ХАИ, 2001. – 65 с.

8. Design of Reliable System Based on Dynamic Bayesian Networks and Genetic Algorithm [Электронный ресурс] / Cao Dingzhou, Kan Shaobai, Sun Yu // Reliability and Maintainability Symposium, 23-26 Jan-

uary, 2012. – P. 1 – 6. – Режим доступа: http://www.reliasoft.com/pubs/2012_RAMS_design_of_reliable_system.pdf/ – 20.12.12 y.

9. Методические указания к лабораторной работе Моделирование систем сетевого планирования и управления по курсу "Экономико-математические модели и методы" [Текст] / Т.В. Алесинская. – Таганрог: Изд-во ТРТУ, 1999. – 36 с.

10. Тюрин, С.Ф. Анализ методов обеспечения пассивной отказоустойчивости цифровых устройств и систем [Текст] / С.Ф. Тюрин, О.А. Громов, А.А. Сулейманов, А.В. Греков // Вестн. ПГТУ. Электротехника, информационные технологии, системы управления. –2011. –№ 5.– С. 144-153.

Поступила в редакцию 28.02.2013, рассмотрена на редколлегии 20.03.2013

Рецензент: канд. техн. наук, доцент, доцент каф. математики и естественнонаучных дисциплин Аляев Юрий Александрович, Пермский филиал Российской Академии народного хозяйства и государственной службы при Президенте Российской Федерации, Пермь.

МЕТОДИКА СИНТЕЗУ СТРУКТУРНОЇ СХЕМИ НАДІЙНОСТІ СИСТЕМИ ІЗ ЗАСТОСУВАННЯМ МОДИФІКОВАНОГО ГРАДІЄНТУ У ПРОЦЕДУРІ НАЙШВИДШОГО СПУСКУ

А.Р. Яматов, С.А. Плесовських, С.Ф. Тюрін

Запропонована нова методика застосування модифікованого градієнта процедури найшвидшого спуску для побудови структурної схеми надійності системи. Проведено порівняльний аналіз з даними отриманими іншими методами. Визначено додаткові етапи до методу найшвидшого спуску підвищують якість ССН системи, а в більшості випадків призводять ССН системи до оптимуму. Проведено дослідження ефективності методів із застосуванням процедури найшвидшого спуску і динамічного програмування при синтезі структурної схеми надійності системи. Застосування запропонованого методу, а також процедури адаптації градієнта, забезпечує істотне зниження інформаційної складності алгоритму пошуку оптимального рішення.

Ключові слова: найшвидшого спуску, модифіковані градієнт, динамічне програмування, проектування структурної схеми надійності, оптимізація, резервування.

МЕТОДИКА СИНТЕЗА СТРУКТУРНОЙ СХЕМЫ НАДЁЖНОСТИ СИСТЕМЫ С ПРИМЕНЕНИЕМ МОДИФИЦИРОВАННОГО ГРАДИЕНТА В ПРОЦЕДУРЕ НАИСКОРЕЙШЕГО СПУСКА

А.Р. Яматов, С.А. Плесовских, С.Ф. Тюрин

Предложена новая методика применения модифицированного градиента процедуры наискорейшего спуска для построения структурной схемы надежности системы. Проведен сравнительный анализ с данными полученными другими методами. Проведено исследование эффективности методов с применением процедуры наискорейшего спуска и динамического программирования при синтезе структурной схемы надежности системы. Определены дополнительные этапы к методу наискорейшего спуска повышающие качество ССН системы, а в большинстве случаев приводящие ССН системы к оптимуму. Применение предлагаемого метода, а также процедуры адаптации градиента, обеспечивает существенное снижение информационной сложности алгоритма поиска оптимального решения.

Ключевые слова: наискорейший спуск, модифицированные градиент, динамическое программирование, проектирование структурной схемы надежности, оптимизация, резервирование.

Яматов Айдар Русланович – старший преподаватель кафедры ПОВТ и АС Пермского военного института внутренних войск МВД России, Пермь, Россия, e-mail: yamatov@mail.ru.

Плесовских Сергей Андреевич – курсант факультета АСУ ПВИ ВВ МВД России, Пермь, Россия.

Тюрин Сергей Феофентович – д-р техн. наук, профессор, профессор кафедры «Автоматика и телемеханика» ПНИПУ, Пермь, Россия.