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BIRNBAUM IMPORTANCE FOR ESTIMATION OF MULTI-STATE AND BINARY-STATE SYSTEMS

There exist two different types of mathematical models that are widely used in reliability analysis: Binary-State System (BSS) and Multi-State System (MSS). Almost every system consists of more than one component. The importance of individual components for the system can be quantified by Importance Measures (IM). One of the most commonly used IM is the Birnbaum Importance (BI). The BI has been defined for the BSS. Development of this measure for the MSS is considered in this paper. New methodological approach based on the use of the Logical Differential calculus is used. The application of this approach allows definition of new equations for computation of the BI for system with series and parallel structure.

Keywords: reliability, importance measure, birnbaum importance, multi-state system, logical differential calculus.

Introduction

Reliability is one of the main characteristics of many systems. There are two mathematical models for the interpretation of an initial system that are named a Binary-State System (BSS) and a Multi-State System (MSS) [1]. The BSS is used for mathematical representation of system with two states: failure/unavailable (indicated as 0) and functioning/available (indicated as 1). The MSS allows defining more than two states (performance levels) of the system: from complete failure (represented by 0) to perfect functioning (indicated as $M-1$), where M is a number of the system performance levels. Every MSS component can take some states too.

Different methods and tools are used for investigation and quantification of the system reliability. Importance analysis is part of reliability engineering. This analysis allows the investigation of the influence of a system component state change to the system performance level. Importance Measures (IMs) are used for the quantification in the importance analysis. IMs permits to investigate different aspects of the system performance level change caused by some component state change [2].

One of the most often used IMs is the Birnbaum Importance [3, 4]. This measure was defined in [3] for the BSS. The MSS Birnbaum Importance has specific aspects in interpretation and calculation. In this paper, the common mathematical approach for calculation of this measure for BSS and MSS, which is based on the Logical Differential Calculus, is proposed.

1. Mathematical Background

Consider the MSS of n components that has M performance levels from 0 to $M-1$. The i -th system

component ($i = 1, 2, \dots, n$) has M_i states that are changed from 0 to M_i-1 . The correlation between system performance level and components states is defined by the Structure Function (SF) [1, 5]:

$$\varphi(\mathbf{x}) : \{0, \dots, M_1-1\} \times \dots \times \{0, \dots, M_n-1\} \rightarrow \{0, \dots, M-1\}, \quad (1)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)$ is a vector of system components states, i.e. state vector.

Note, the equation (1) can be used for definition of the SF of the BSS if $M_1 = M_2 = \dots = M_n = M = 2$:

$$\varphi(\mathbf{x}) : \{0, 1\}^n \rightarrow \{0, 1\}. \quad (2)$$

Every system component is characterized by probability of the component state [1]:

$$p_{i,s} = \Pr\{x_i = s\}, \quad (3)$$

where $p_{i,s}$ ($s = 0, 1, \dots, M_i-1$) is the probability that the i -th component is in state s .

Each performance level of the MSS is estimated by the probability that is calculated by the MSS SF [5]:

$$R(j) = \Pr\{\varphi(\mathbf{x}) = j\}, \quad j = 1, 2, \dots, M-1. \quad (4)$$

There is other interpretation of the probability of the MSS performance level [1, 6]:

$$R(j) = \Pr\{\varphi(\mathbf{x}) \geq j\}, \quad j = 1, 2, \dots, M-1. \quad (5)$$

In the case of the BSS, the probabilities (4) and (5) are equal and are named as system reliability:

$$R = \Pr\{\varphi(\mathbf{x}) = 1\}. \quad (6)$$

The system unreliability (for the MSS and BSS) is calculated as follows:

$$F = \Pr\{\varphi(\mathbf{x}) = 0\}. \quad (7)$$

There exists a special class of MSSs, which SF meets the condition that $M_1 = M_2 = \dots = M_n = M$. This

MSS SF is interpreted as the Multiple-Valued Logic (MVL) function [7]:

$$\varphi(\mathbf{x}): \{0, \dots, M-1\}^n \rightarrow \{0, \dots, M-1\}. \quad (8)$$

This assumption permits to use and adapt some methods of the MVL for the MSS reliability estimation. The Logical Differential Calculus is proposed for importance analysis of the MSS in [5, 7] in particular. This MVL tool is used for analysis of the MVL function depending on changes of its variables. Therefore logic derivatives can be used to evaluate the influence of the system component state change on performance level of the MSS. This methodology has been firstly considered in [8, 9].

There are different types of logical derivatives in MVL [7, 9]. The Direct Partial Logic Derivative (DPLD) of the MVL function is one of these types [10]. These derivatives reflect the change in the value of the underlying function when the values of variables change. DPLD $\frac{\partial \varphi(j \rightarrow h)}{\partial x_i(s \rightarrow s-1)}$ of the function $\varphi(\mathbf{x})$ of n variables with respect to variable x_i reflects the fact of changing of the function from j to h when the value of variable x_i is changing from s to $s-1$:

$$\begin{aligned} \frac{\partial \varphi(j \rightarrow h)}{\partial x_i(s \rightarrow s-1)} &= \\ &= \begin{cases} 1 & \text{if } \varphi(s_i, \mathbf{x}) = j \text{ and } \varphi((s-1)_i, \mathbf{x}) = h \\ 0 & \text{other} \end{cases}, \end{aligned} \quad (9)$$

where $\varphi(s_i, \mathbf{x})$ is a value of structure function for state vector $(x_1, \dots, x_{i-1}, s, x_{i+1}, \dots, x_n)$, $s \in \{1, 2, \dots, M_i-1\}$.

The DPLD of the SF of the BSS is a special case of (9) and the DPLD for the BSS SF with respect to variable x_i is defined as [11]:

$$\frac{\partial \varphi(j \rightarrow \bar{j})}{\partial x_i(s \rightarrow s)} = \{\varphi(s_i, \mathbf{x}) = j\} \wedge \{\varphi(\bar{s}_i, \mathbf{x}) = \bar{j}\}. \quad (10)$$

In this paper, the coherent system is considered. The coherent system SF satisfies the following assumptions [1]:

- (a) every component is relevant to the system performance,
- (b) the SF (1), (2) and (8) of the system is non-decreasing.

2. The Birnbaum Importance for the BSS

2.1. The Definition of the Birnbaum Importance

The Birnbaum Importance (BI) is a measure that characterizes the influence of the i -th system component on the BSS functioning and the criticality of this component to BSS functioning. It represents loss in the system functioning when the i -th component was failed.

The BI of the i -th component is probabilistic measure and has been defined as [3]:

$$\begin{aligned} I_B(x_i) &= \Pr\{\varphi(\mathbf{x}) = 1 \mid x_i = 1\} - \Pr\{\varphi(\mathbf{x}) = 1 \mid x_i = 0\} = \\ &= \Pr\{\varphi(\mathbf{x}) = 0 \mid x_i = 0\} - \Pr\{\varphi(\mathbf{x}) = 0 \mid x_i = 1\}. \end{aligned} \quad (11)$$

The BI can be calculated by other equations that have been defined in [12, 13]:

$$I_B(x_i) = \frac{\partial R}{\partial p_{i,1}}, \quad (12)$$

$$I_B(x_i) = \Pr\{\varphi(1_i, \mathbf{x}) - \varphi(0_i, \mathbf{x}) = 1\}, \quad (13)$$

$$I_B(x_i) = E(\varphi(1_i, \mathbf{x}) - \varphi(0_i, \mathbf{x})). \quad (14)$$

According to (12), the BI measures the sensitivity of the system reliability to the change in the reliability of the component i . So, if the BI value is large, then a small change in the component reliability will result in a comparatively large change in the system reliability [14]. The BI (13) is calculated as the probability that the i -th component is critical for the system [12].

According to (14), the BI indicates the expected decrease in the system state caused by the failure of the component i .

The interpretation of the BI based on the DPLD (10) has been considered in [11]: the BI of the i -th system component is calculated as the probability of the nonzero values of the DPLD with respect to the i -th variable x_i :

$$I_B(x_i) = \Pr\left\{\frac{\partial \varphi(1 \rightarrow 0)}{\partial x_i(1 \rightarrow 0)} = 1\right\}. \quad (15)$$

1.1. The Birnbaum Importance for the Series and Parallel BSS

There are two typical structures of systems in reliability engineering (Fig. 1.): series and parallel. These systems have SF with the regularity.

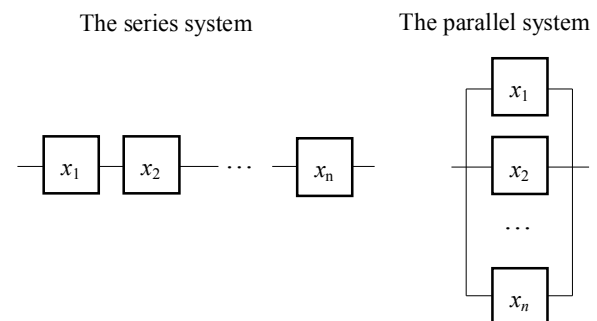


Fig. 1. The series and parallel system

The SF of the series BSS is defined as:

$$\varphi(\mathbf{x}) = x_1 \text{ AND } x_2 \text{ AND } \dots \text{ AND } x_n \quad (16)$$

and has only one nonzero value that agrees with the vector of components states $\mathbf{x} = (1, 1, \dots, 1)$. DPLD $\partial\varphi(1 \rightarrow 0)/\partial x_i(1 \rightarrow 0)$ of this SF, with respect to any variable, has only one nonzero value:

$$(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) = (1, \dots, 1, 1, \dots, 1). \quad (17)$$

Therefore the BI of the series BSS according to (15) and (17) is calculated as follows:

$$I_B(x_i) = \Pr \left\{ \frac{\partial\varphi(1 \rightarrow 0)}{\partial x_i(1 \rightarrow 0)} = 1 \right\} = p_{1,1} \dots p_{i-1,1} p_{i+1,1} \dots p_{n,1}. \quad (18)$$

The SF of the parallel BSS is defined as:

$$\varphi(\mathbf{x}) = x_1 \text{ OR } x_2 \text{ OR } \dots \text{ OR } x_n \quad (19)$$

and has only one zero value that agrees with state vector $\mathbf{x} = (0, 0, \dots, 0)$. DPLD $\partial\varphi(1 \rightarrow 0)/\partial x_i(1 \rightarrow 0)$ of the SF with respect to the i -th variable has only one nonzero value:

$$(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n) = (0, \dots, 0, 0, \dots, 0). \quad (20)$$

Therefore the BI of the parallel BSS according to (15) and (20) is calculated as

$$I_B(x_i) = \Pr \left\{ \frac{\partial\varphi(1 \rightarrow 0)}{\partial x_i(1 \rightarrow 0)} = 1 \right\} = p_{1,0} \dots p_{i-1,0} p_{i+1,0} \dots p_{n,0}. \quad (21)$$

2.3. The Hand Calculation Example

For example, consider the BSS of three components ($n = 3$) in two variants: with series and parallel structure. The values of components states probabilities of such systems are in table 1. Compute the BIs $I_B(x_i)$ of such systems.

Table 1

Components states probabilities

Component	State	
	0	1
1	0,2	0,8
2	0,3	0,7
3	0,1	0,9

The BI for the series BSS of the i -th system component is calculated by (18) as the multiplication of probabilities of system components functioning $p_{i,1}$ ($i = 1, \dots, i-1, i+1, \dots, n$) (table 2). The BI of system component i for the parallel BSS is calculated similarly by (21) and values of these measures are in Table 2 too.

Therefore the application of the DPLD in the importance analysis permits to get new equations for com-

putation of BIs for the series and parallel BSS. The simple algorithms for calculation of these BIs are implemented based on the equation (18) and (21).

Table 2

BIs for the series and parallel BSS

Component	The Birnbaum Importance	
	The series system	The parallel system
1	0,63	0,03
2	0,72	0,02
3	0,56	0,06

3. The Birnbaum Importance for the MSS

3.1. The Definition of the Birnbaum Importance

The BIs for the MSS have some interpretations and definitions, which are equivalent in the case of BSSs but not in the case of MSSs. It is caused by the ambiguity in generalization of the IMs for the MSS from the BSS. Consider some of the possible interpretations of the BI for the MSS below.

The definition of the BI that is named s, r -BI is proposed in [15]. The s, r -BI $I_{B-s,r}(x_i, j)$ for the MSS is the probability that the transition of the i -th component from state s to r causes the system reliability degradation [15]:

$$I_{B-s,r}(x_i, j) = \Pr \{ \varphi(s_i, \mathbf{x}) \geq j \text{ AND } \varphi(r_i, \mathbf{x}) < j \}, \quad (22)$$

or special case:

$$I_{B-s,r}(x_i, j) = \Pr \{ \varphi(s_i, \mathbf{x}) = j \text{ AND } \varphi(r_i, \mathbf{x}) < j \}. \quad (23)$$

In the paper [16], the BI (23) was modified as the system degradation depending on one component state:

$$I_B(s_i, j) = \Pr \{ \varphi(s_i, \mathbf{x}) = j \text{ AND } \varphi((s-1)_i, \mathbf{x}) < j \}. \quad (24)$$

The BI (24) in terms of the DPLD (9) can be defined as:

$$I_B(s_i, j) = \Pr \left\{ \sum_{h=0}^{j-1} \left(\frac{\partial\varphi(j \rightarrow h)}{\partial x_i(s \rightarrow s-1)} \right) = 1 \right\}. \quad (25)$$

The BIs (24) and (25) are defined for the i -th system component state s and the MSS performance level j . The BI (25) is probability of the MSS performance level change from j to $j-1$ if the component state degrades from s to $s-1$. The BI $I_B(s_i, j)$ for all relevant states of the i -th component ($s = 1, 2, \dots, M_i - 1$) and for the MSS performance level j is calculated as:

$$I_B(x_i, j) = \frac{\sum_{s=1}^{M_i-1} I_B(s_i, j)}{M_i - 1}. \quad (26)$$

The BI $I_B(x_i, j)$ quantifies the probability that the i -th component is critical for system state j .

Next generalization of the BI $I_B(s_i, j)$ is $I_B(s_i)$ as the probability that state s of the i -th component is critical for the MSS:

$$I_B(s_i) = \sum_{j=1}^{M-1} I_B(s_i, j). \quad (27)$$

Finally, the overall BI of the i -th component can be introduced as:

$$I_B(x_i) = \frac{\sum_{s=1}^{M_i-1} I_B(s_i)}{M_i - 1}. \quad (28)$$

3.2. The Birnbaum Importance for series and parallel MSS

Series and parallel systems (shown on Fig. 1.), can be defined for the MSS too. In what follows, we assume that $M_1 = M_2 = \dots = M_n = M$, i.e. the SFs of series and parallel systems are MVL functions.

The SF of the series MSS can be defined as:

$$\varphi(\mathbf{x}) = \text{MIN}(x_1, x_2, \dots, x_n). \quad (29)$$

In this case, only one DPLD contains nonzero values, i.e. DPLD $\partial\varphi(s \rightarrow s-1)/\partial x_i(s \rightarrow s-1)$. This DPLD is nonzero for state vectors $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$, which satisfy the following condition:

$$x_l \geq s \text{ for all } l = 1, 2, \dots, i-1, i+1, \dots, n. \quad (30)$$

Then, the BI $I_B(s_i, j)$ of this MSS according to (25) and (30) is calculated as:

$$I_B(s_i, j) = \Pr \left\{ \sum_{h=0}^{j-1} \frac{\partial\varphi(j \rightarrow h)}{\partial x_i(s \rightarrow s-1)} = 1 \right\} = \begin{cases} \prod_{l=1, l \neq i}^n \left(\sum_{w=s}^{M-1} p_{l,w} \right) & \text{if } j = s; \\ 0 & \text{if } j \neq s. \end{cases} \quad (31)$$

From (31), it is clear that the BI $I_B(s_i)$ (27) can be computed in the following manner:

$$I_B(s_i) = \Pr \left\{ \frac{\partial\varphi(s \rightarrow s-1)}{\partial x_i(s \rightarrow s-1)} = 1 \right\} = \prod_{l=1, l \neq i}^n \left(\sum_{w=s}^{M-1} p_{l,w} \right) \quad (32)$$

and the BI $I_B(x_i)$ (28) is:

$$I_B(x_i) = \frac{\sum_{s=1}^{M-1} \Pr \left\{ \frac{\partial\varphi(s \rightarrow s-1)}{\partial x_i(s \rightarrow s-1)} = 1 \right\}}{M-1} = \frac{1}{M-1} \sum_{s=1}^{M-1} \left(\prod_{l=1, l \neq i}^n \left(\sum_{w=s}^{M-1} p_{l,w} \right) \right). \quad (33)$$

The SF of the parallel MSS is defined as:

$$\varphi(\mathbf{x}) = \text{MAX}(x_1, x_2, \dots, x_n). \quad (34)$$

There is one DPLD for the parallel MSS, which has nonzero values, i.e.

$$\partial\varphi(s \rightarrow s-1)/\partial x_i(s \rightarrow s-1).$$

This DPLD has nonzero values for state vectors $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$, which meet the following condition:

$$x_l < s \text{ for all } l = 1, 2, \dots, i-1, i+1, \dots, n. \quad (35)$$

The BI $I_B(s_i, j)$ of the parallel MSS is calculated according to (25) and (35):

$$I_B(s_i, j) = \Pr \left\{ \sum_{h=0}^{j-1} \frac{\partial\varphi(j \rightarrow h)}{\partial x_i(s \rightarrow s-1)} = 1 \right\} = \begin{cases} \prod_{l=1, l \neq i}^n \left(\sum_{w=0}^{s-1} p_{l,w} \right) & \text{if } j = s; \\ 0 & \text{if } j \neq s. \end{cases} \quad (36)$$

The BI $I_B(s_i)$ (27) can be calculated as:

$$I_B(s_i) = \Pr \left\{ \frac{\partial\varphi(s \rightarrow s-1)}{\partial x_i(s \rightarrow s-1)} = 1 \right\} = \prod_{l=1, l \neq i}^n \left(\sum_{w=0}^{s-1} p_{l,w} \right) \quad (37)$$

and the BI $I_B(x_i)$ (28) is defined as:

$$I_B(x_i) = \frac{\sum_{s=1}^{M-1} \Pr \left\{ \frac{\partial\varphi(s \rightarrow s-1)}{\partial x_i(s \rightarrow s-1)} = 1 \right\}}{M-1} = \frac{1}{M-1} \sum_{s=1}^{M-1} \left(\prod_{l=1, l \neq i}^n \left(\sum_{w=0}^{s-1} p_{l,w} \right) \right). \quad (38)$$

So, using DPLDs, we have derived quite simple formulae for computation of the BI of series and parallel MSSs, which are defined by MIN and MAX MVL-functions.

3.3. The Hand Calculation Example

Consider the MSS of three components ($n = 3$) in two variants: with series (29) and parallel (34) structure. Every component can be in one of 3 states ($M_1 = M_2 = M_3 = M = 3$), and values of components states probabilities of these systems are in table 3.

Table 3
Components states probabilities

Component	State		
	0	1	2
1	0,1	0,1	0,8
2	0,3	0,1	0,6
3	0,1	0,2	0,7

The BIs for the series MSS are calculated by (33) and the BI of the i -th system component of the parallel MSS is calculated by (38) (table 4).

Table 4
BIs for the series and parallel MSS

Component	The Birnbaum Importance	
	The series system	The parallel system
1	0.525	0.075
2	0.685	0.035
3	0.555	0.055

Conclusion

Reliability is very important part of many systems. One of the essential problems of reliability analysis is the estimation of the influence of system components on the system performance.

This influence can be measured by IMs. One of the basic IMs is the BI.

In this paper, the calculation of the BI, using DPLDs, has been considered.

We have shown that Logical Differential Calculus can be very successfully used for derivation of formulae for computation of the BI for various types of systems, i.e. this calculus can be used in the case of BSSs and MSSs too.

In this paper, we have studied only coherent systems. In the future work it will be very interesting to consider using the technique of DPLDs in the Importance Analysis of non-coherent systems.

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ОЦІНКА БІРНБАУМА ДЛЯ АНАЛІЗУ НАДІЙНОСТІ СИСТЕМИ З ДЕКІЛЬКОМА І ДВОМА РІВНЯМИ ПРАЦЕЗДАТНОСТІ

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У теорії надійності використовуються дві математичні моделі, що описують два або декілька рівнів працездатності системи. Як правило, такі системи складаються більш ніж з одного компоненту, і тому, важливим завданням є оцінка ступеню впливу кожного з компонентів системи на її працездатність. Ця оцінка здійснюється, наприклад, за допомогою оцінок значущості. Найчастіше на практиці використовується оцінка Бірнбаума. Спочатку ця оцінка була визначена для систем з двома рівнями працездатності. У даній роботі пропонується узагальнення цієї оцінки для випадку систем з декількома рівнями працездатності, а також розглядається новий підхід до її обчислення на основі математичного апарату логічного диференціального числення. Використання цього підходу дозволяє отримати оцінки Бірнбаума для послідовних і паралельних структур.

Ключові слова: надійність, оцінки значущості елементів, оцінка Бірнбаума, системи з декількома рівнями працездатності, логічне диференціальне числення.

ОЦЕНКА БИРНБАУМА ДЛЯ АНАЛИЗА НАДЕЖНОСТИ СИСТЕМЫ С НЕСКОЛЬКИМИ И ДВУМЯ УРОВНЯМИ РАБОТОСПОСОБНОСТИ

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В теории надежности используются две математические модели, описывающие два или несколько уровней работоспособности системы. Как правило, такие системы состоят более чем из одного компонента, и поэтому, важной задачей является оценка степени влияния каждого из компонентов системы на ее работоспособность. Эта оценка осуществляется, например, с помощью оценок значимости. Наиболее часто на практике используется оценка Бирнбаума. Первоначально эта оценка была определена для систем с двумя уровнями работоспособности. В данной работе предлагается обобщение этой оценки для случая систем с несколькими уровнями работоспособности, а также рассматривается новый подход к ее вычислению на основе математического аппарата логического дифференциального исчисления. Использование этого подхода позволяет получить оценки Бирнбаума для последовательных и параллельных структур.

Ключевые слова: надежность, оценки значимости элементов, оценка Бирнбаума, системы с несколькими уровнями работоспособности, логическое дифференциальное исчисление.

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