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DETERMINATION OF THE EVOLUTIONARY CRITICAL INFRASTRUCTURES SYSTEM CHARACTERISTICS ON THE BASIS OF THE SEMI-MARKOV THEORY

Definition of evolutionary processes for critical infrastructures are given in the work. Analytical model of evolution are developed, on the basis of semi-Markov theory. Formula for calculation of basic system characteristic of evolution (mathematical expectation of time needed for evolution) are purposed. Some minuses of analytical approach for solving of the problem of critical infrastructures evolution researching are listed. Simulation modeling is purposed as an alternative solving for the problem.

Keywords: *evolution of Critical Infrastructures, concept of "potentional", semi-Markov Model, simulation modeling.*

Introduction

There is no universal definition of evolution process as phenomenon. As well as there is no universal model of evolution of critical infrastructure.

In this work we concretize term "evolution" as a following process.

Critical infrastructure consists of systems. Normal functioning of critical infrastructure is maintaining of balance between potentials of infrastructure and consumed resources.

If the amount of consumed resources changes, it's necessary to make changes in subsystems in order to keep balance in the whole system.

1. Definition of evolutionary processes

Evolution of critical infrastructure subsystems is obtaining or quality changes in some characteristics over the certain time. On every stage of evolution system improve its quality characteristics [1].

If subsystem of critical infrastructure evolves over the time, less then certain valid time, then process of evolution is successful. Otherwise subsystem should be returned to last useable state and evolve by another way.

Successful evolution of critical infrastructure take place only when all subsystems evolve successfully while balance between potential of infrastructure and consumed resources.

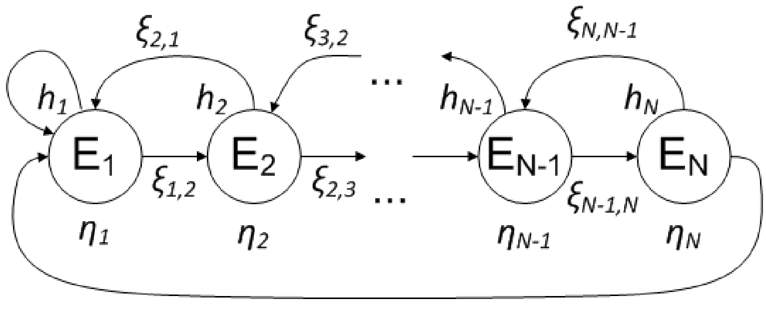

Fig. 1. Stages of CI's evolution

The evolution of critical infrastructure goes through N-1 stages, which are defined on phase space $E = \{1,2,\ldots,N-1,N\}$ (fig 1):

η_i – time needed for the evolution of critical infrastructure on stage with number i;

$\eta_i = F_i(x)$, where F_i – distribution function of random value;

h_i – maximum time allowed for evolution on i^{th} stage.

System is evolving when $\eta_i \le h_i$, otherwise system returns to previous stage.

$\xi_{i,j}$– probability of transition of critical infrastructure from stage i into stage j.

$$\xi_{k,k+1} + \xi_{k,k-1} = 1$$

2. Analytical model of CI's evolution

The evolution of critical infrastructure can be describes by following sequences: $\{\xi_{k,k}\}$ and $\{\eta_k\}$. Mathematical model of critical infrastructures evolution may be built on the basis of these sequences[2].

Transition probabilities in Markov Chain:

$$\begin{cases} P\{\zeta_{n+1} = k+1 \mid \zeta_n = k\} = P\{\eta_k \le h_k\} = F_k(h_k); \\ P\{\zeta_{n+1} = k-1 \mid \zeta_n = k\} = P\{\eta_k > h_k\} = \\ = 1 - F_k(h_k) = \overline{F}_k(h_k); \\ P\{\zeta_{n+1} = 1 \mid \zeta_n = 1\} = P\{\eta_1 > h_1\} = F_1(h_1); \\ P\{\zeta_{n+1} = 1 \mid \zeta_n = N\} = 1. \end{cases}$$

Other transition probabilities are 0. θ_k – time, that system spent in stage with number k.

$$\begin{cases} \theta_k = \min\{\eta_k, h_h\} = \eta_k \wedge h_h, k = \overline{1, N-1}; \\ \theta_N = 0. \end{cases}$$

Time, that system spent in k^{-th} state, if system will transit to l^{-th} state:

$$\theta_{k,k+1} = \eta_k,\ \theta_{k,k-1} = h_k$$
$$\theta_{1,1} = h_1$$
$$\theta_{N,1} = 0$$

Other transition times are 0. $M_{k,l}$ – time of transition of system from k^{-th} state to l^{-th} one.

$$M_{k,l} = h_k,\ \text{if } l = k-1; k = \overline{1, N\text{-}1};$$
$$M_{k,l} = M\{\eta_k\},\ \text{if } l = k+1; k = \overline{2, N\text{-}1};$$
$$M_{1,1} = h_1;$$
$$M_{N,1} = 0.$$

$t_{1,N}$ – expectation of the time of systems transition from 1^{st} state to N^{th} state is the main system characteristic of evolutionary critical infrastructure.

Analytical mathematical model of evolutionary system:

$$\begin{cases} t_{1,N} - (P_{1,1}t_{1,N} + P_{1,2}t_{2,N}) = \\ = P_{1,1}M_{1,1} + P_{1,2}M_{1,2} + P_{1,N}M_{1,N}; \\ t_{2,N} - (P_{2,1}t_{1,N} + P_{2,3}t_{3,N}) = \\ = P_{2,1}M_{2,1} + P_{2,3}M_{2,3} + P_{2,N}M_{2,N}; \\ \dots \\ t_{i,N} - (P_{i,i-1}t_{i-1,N} + P_{i,i+1}t_{i+1,N}) = \\ = P_{i,i-1}M_{i,i-1} + P_{i,i+1}M_{i,i+1} + P_{i,N}M_{i,N}; \\ \dots \\ t_{N-1,N} - P_{N-1,N-2}t_{N-2,N} = \\ = P_{N-1,N-2}M_{N-1,N-2} + P_{N-1,N}M_{N-1,N}. \end{cases}$$

Also we can show following system:

$$\begin{cases} t_{1,N}(1 - \overline{F_1}(h_1)) - t_{2,N}F_1(h_1) = \\ = \overline{F_1}(h_1)h_1 + F_1(h_1)M\{\eta_1\}; \\ t_{2,N} - (t_{1,N}\overline{F}_2(h_2) + t_{3,N}F_2(h_2)) = \\ = \overline{F_2}(h_2)h_2 + F_2(h_2)M\{\eta_2\}; \\ \dots \\ t_{i,N} - (t_{i-1,N}\overline{F}_i(h_i) + t_{i+1,N}F_i(h_i)) = \\ = \overline{F_i}(h_i)h_i + F_i(h_i)M\{\eta_i\}; \\ \dots \\ t_{N-1,N} - t_{N-2,N}\overline{F_{N-1}}(h_{N-1}) = \\ = \overline{F_{N-1}}(h_{N-1})h_{N-1} + F_{N-1}(h_{N-1})M\{\eta_{N-1}\}. \end{cases}$$

We assume following notation:

$$h_i - F_i(h_i)(h_i - M\{\eta_i\}) = b_i,\ F_i(h_i) = F_i$$

Then previous system can be written as:

$$\begin{cases} t_{1,N}F_1 - t_{2,N}F_1 = b_1; \\ -t_{1,N}\overline{F_2} + t_{2,N} - t_{3,N}F_2 = b_2; \\ \dots \\ -t_{i-1,N}\overline{F}_i + t_{i,N} - t_{i+1,N}F_i = b_i; \\ \dots \\ -t_{N-2,N}\overline{F}_{N-1} + t_{N-1,N} = b_{N-1}. \end{cases}$$

Determinant of previous system

$$\Delta_N = \prod_{k=1}^{N-1} F_k\ .$$

Let's show additional determinant $\Delta_{1,N+1}$, for calculating $t_{1,N.}$

$$\Delta_{1,N+1} =$$
$$= \begin{vmatrix} b_1 & -F_1 & 0 & \dots & 0 & 0 & 0 & 0 \\ b_2 & 1 & -F_2 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ b_{N-2} & 0 & 0 & \dots & -\overline{F_{N-2}} & 1 & -F_{N-2} & 0 \\ b_{N-1} & 0 & 0 & \dots & 0 & -\overline{F_{N-1}} & 1 & -F_{N-1} \\ b_N & 0 & 0 & \dots & 0 & 0 & -\overline{F_N} & 1 \end{vmatrix},$$

Let's define determinant $\Delta_{1,N+1}$ as H_{N+1} and after it's calculation we'll take following formula:

$$H_{N+1} =$$
$$= H_N - F_{N-1}\overline{F_N}H_{N-1} + b_N\prod_{k=1}^{N-1}F_k.$$

We have:

$$H_2 = b_1,$$

$$H_3 = b_1 + b_2 F_1 = \sum_{i=1}^{2} b_i \prod_{k=1}^{i-1} F_i;$$

$$H_4 = b_1 + b_2 F_1 + b_3 F_1 F_2 \overline{F}_3 =$$

$$-\sum_{i=1}^{3} b_i \prod_{k=1}^{i-1} F_i - \sum_{i=1}^{1} b_i \sum_{l=2}^{2} F_l \overline{F}_{l+1}.$$

We assume:

$$H_N = \sum_{i=1}^{N-1} b_i \prod_{k=1}^{i-1} F_k - \sum_{i=1}^{N-3} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{n-2} F_l \overline{F}_{l+1} +$$

$$+\sum_{i=1}^{N-5} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-4} F_l \overline{F}_{l+1} \sum_{m=l+2}^{N-2} F_m \overline{F}_{m+1} - \ldots$$

$$H_{N-1} = \sum_{i=1}^{N-2} b_i \prod_{k=1}^{i-1} F_k -$$

$$-\sum_{i=1}^{N-4} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-3} F_l \overline{F}_{l+1} +$$

$$+\sum_{i=1}^{N-6} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i-1}^{N-5} F_l \overline{F}_{l+1} \sum_{m=l+2}^{N-3} F_m \overline{F}_{m+1} - \ldots$$

Let's prove following formula:

$$H_{N+1} = \sum_{i=1}^{N-1} b_i \prod_{k=1}^{i-1} F_k - \sum_{i=1}^{N-3} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-2} F_l \overline{F}_{l+1} +$$

$$+\sum_{i=1}^{N-5} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-4} F_l \overline{F}_{l+1} \sum_{m=l+2}^{N-2} F_m \overline{F}_{m+1} - \ldots$$

$$\ldots - F_{N-1}\overline{F}_N \sum_{i=1}^{N-2} b_i \prod_{k=1}^{i-1} F_k + F_{N-1}\overline{F}_N \sum_{i=1}^{N-4} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-3} F_l \overline{F}_{l+1} -$$

$$-F_{N-1}\overline{F}_N \sum_{i=1}^{N-6} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-5} F_l \overline{F}_{l+1} \times$$

$$\times \sum_{m=l+2}^{N-3} F_m \overline{F}_{m+1} + \ldots + b_N \prod_{k=1}^{N-1} F_k.$$

Group the terms of formula with the same number of factors:

$$H_{N+1} = \sum_{i=1}^{n} b_i \prod_{k=1}^{i-1} F_k -$$

$$-(\sum_{i=1}^{n-3} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-2} F_l \overline{F}_{l+1} + F_{N-1}\overline{F}_N \sum_{i=1}^{N-2} b_i \prod_{k=1}^{i-1} F_k) +$$

$$+(\sum_{i=1}^{N-6} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-4} F_l \overline{F}_{l+1} \sum_{m=l+2}^{N-2} F_m \overline{F}_{m+1} +$$

$$+F_{N-1}\overline{F}_N \sum_{i=1}^{N-4} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i}^{N-3} F_m \overline{F}_{m+1}).$$

Term in first brackets convert into:

$$\sum_{i=1}^{N-3} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-2} F_l \overline{F}_{l+1} +$$

$$= \sum_{i=1}^{N-3} b_i \prod_{k=1}^{i-1} F_k F_{N-1}\overline{F}_N + b_{N-2} \prod_{k=1}^{N-3} F_k F_{N-1}\overline{F}_N =$$

$$= \sum_{i=1}^{N-3} b_i \prod_{k=1}^{i-1} F_k (\sum_{l=i-1}^{N-2} F_l \overline{F}_{l+1} + F_{N-1}\overline{F}_N) +$$

$$+b_{N-2} \prod_{k=1}^{N-3} F_k F_{N-1}\overline{F}_N = \sum_{i=1}^{N-2} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-1} F_l \overline{F}_{l+1}.$$

The same way let's group terms into the second brackets. Finally we take following formula:

$$H_{N+1} =$$

$$= \sum_{i=1}^{N} b_i \prod_{k=1}^{i-1} F_k - \sum_{i=1}^{N-2} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-1} F_l \overline{F}_{l+1} +$$

$$+\sum_{i=1}^{N-4} b_i \prod_{k=1}^{i-1} F_k \sum_{l=i+1}^{N-3} F_l \overline{F}_{l+1} \sum_{m=l+2}^{N-1} F_m \overline{F}_{m+1} - \ldots$$

The resulting formula is the same as previously proposed.

Let's take formula for calculating of expectation of the time of evolution from previous one:

$$t_{1,N} = \sum_{i=1}^{N} \frac{b_i}{\prod_{k=i}^{N-1} F_k} - \sum_{i=1}^{N-2} \frac{b_i}{\prod_{k=i}^{N-1} F_k} \sum_{l=i+1}^{N-1} F_l \overline{F}_{l+1} + \sum_{i=1}^{N-4} \frac{b_i}{\prod_{k=i}^{N-1} F_k} \sum_{l=i+1}^{N-3} F_l \overline{F}_{l+1} \sum_{m=l+2}^{N-1} F_m \overline{F}_{m+1} - \sum_{i=1}^{N-6} \frac{b_i}{\prod_{k=i}^{N-1} F_k} \sum_{l=i+1}^{N-5} F_l \overline{F}_{l+1} *$$

$$* \sum_{m=l+2}^{N-3} F_m \overline{F}_{m+1} \sum_{j=m+2}^{N-1} F_j \overline{F}_{j+1} + \ldots$$

3. Example of solving of analytical model

Here is an example of calculating of mean of the critical infrastructure evolution time.

The number of stages in evolution: N-1=5;

Average time of evolution for every stage is distributed as a normal random value with parameters of distribution m=0.5, σ=0.2 (fig. 2).

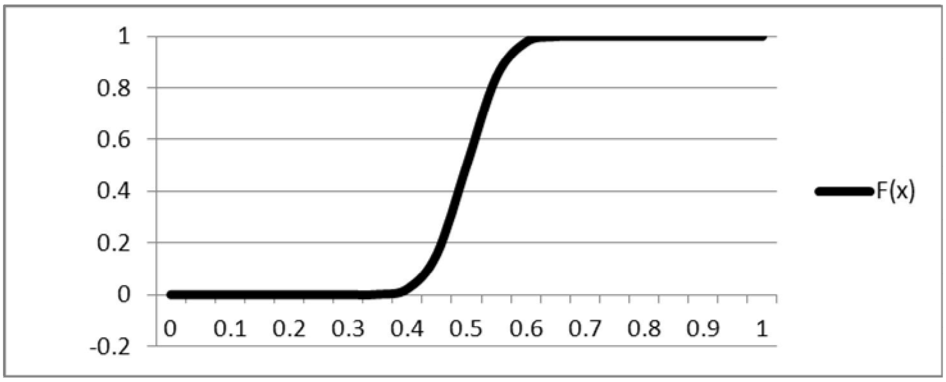

Fig. 2. distribution function of random value x.

Values of maximum allowed time for system evolving on i[th] stage are same and uniform distributed between 0 and 1.

Let's show dependence (fig. 3) of the time of evolution of critical infrastructure from h_i value.

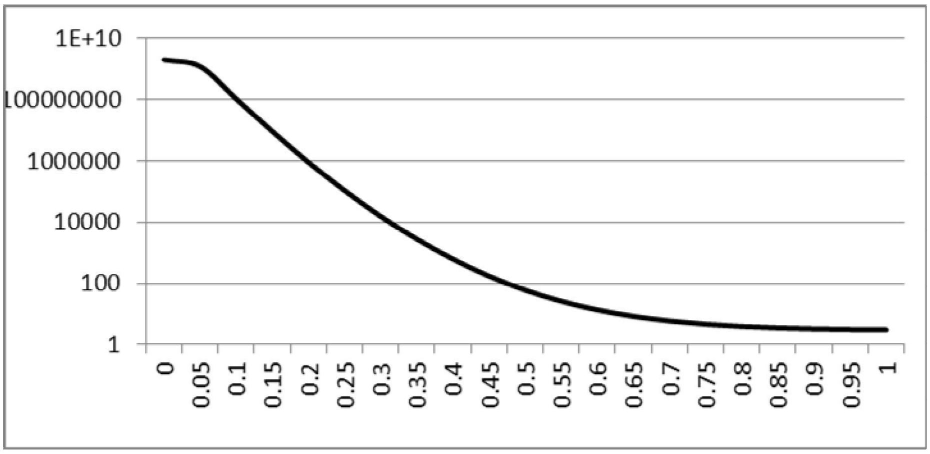

Fig. 3. dependence of the time of evolution ofcritical infrastructure from h_i value.

The obtained dependence clearly demonstrates that the minimum time of evolution is reached at the maximum allowable limit of time for evolution on i[th] stage.

4. Simulation modeling of critical infrastructure evolution

Simulation modeling allows to get the implementation trajectory and time of CI's evolution time for a quite short time period.

Using together simulation modeling and statistical methods allows to find the time of critical infrastructure evolution with a specified error without using of analytical models.

To study behavior of CI under different external conditions, we purpose developed software (simulation model). This software can be used as well as in scientific researches and in process of studding of specialists in CI safety.

Now that software is the first version of the research bench purposed to study the evolution of CI.

It has an hierarchical user interface. With increasing of number of the hierarchical level of user interface amounts of available for user data increases as well, as tools.

User interface on a first hierarchical level allows user to set parameters of stages in CI's evolution, to obtain a trajectory of evolution and basic CI's characteristic – mathematical expectation of time needed for evolving of CI.

User interface (fig 4,5) of a high level allows user to add into model additional program modules and links this simulation model with another models.

On first hierarchical level interface include 2 tabs. First one – designed for a setting model parameters (fig 4). Second – shows results of simulation modeling (fig 5).

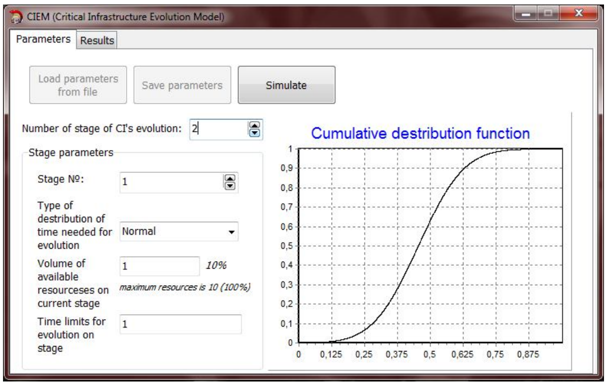

Fig. 4. User interface of CI’s simulation model (Parameters tab)

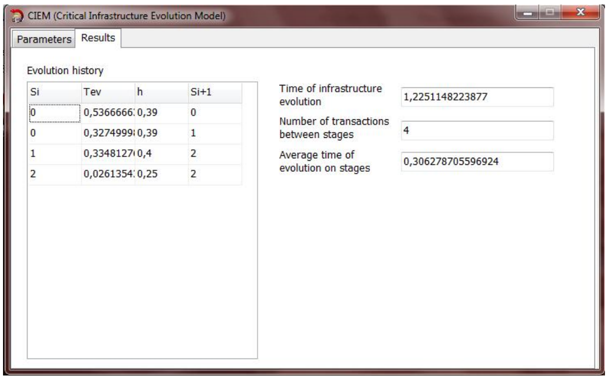

Fig. 5. User interface of CI’s simulation model (Results tab)

To obtain the time of CI evolution user should follow the next scenario:

1. Select tab “Parameters”, if another is selected.
2. Enter the number of stages of CI evolution in field “Number of stages of CI’s evolution”.
3. In the “Stage parameters” must be set:
   - type of distribution;
   - volume of available resources on that stage;
   - time limits for evolution.
4. Procedure on step №3 should be repeated for all stages.
5. Press button “Simulate” for running of modeling and get results on tab “Result”.

Tasks which can be solved using proposed simulation model:

Finding dependence between value of mathematical expectation of time of CI evolution and such parameters as:

- Number of stages of evolution
- Allocation of resources between stages of evolution
- Characteristics of CI, which define evolution time distribution.

Also purposed simulation model can be used for solving optimization tasks such as: “Dynamical allocation of resources between the evolution stages with limitation of CI’s life time and/or length of evolution trajectory”.

Conclusion

It should be noted, that analytical modeling is an exact method of modeling and evolution time calculating for CIs. However, the applicability of this method is limited, due to high complexity of resulting formula for critical infrastructures with big amount of stages of evolution.

In such situations, we may use simulation models of the evolutioning critical infrastructures.

Purposed simulation model can be used for solving optimization tasks such as: “Dynamical allocation of resources between the evolution stages with limitation of CI’s life time and/or length of evolution trajectory”.

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ВИЗНАЧЕННЯ СИСТЕМНИХ ХАРАКТЕРИСТИК ЕВОЛЮЦІЇ КРИТИЧНИХ ІНФРАСТРУКТУР НА ОСНОВІ ТЕОРІЇ НАПІВМАРКОВСЬКИХ ПРОЦЕСІВ

О.В. Скатков, Н.О. Скаткова, В.С. Ловягін

Дається визначення процесу еволюції критичної інфраструктури. У статті надається, розроблена на основі теорії напівмарковських процесів, аналітична модель еволюції критичних інфраструктур. Наводиться формула для розрахунку базових системних характеристик еволюції (математичного очікування часу, необхідного для еволюції. Перерахування деяких недоліки аналітичного підходу до вирішення проблеми дослідження еволюції критично інфраструктур. Запропоновано імітаційну модель еволюції як альтернативне рішення проблеми.

Ключові слова: еволюція критичних інфраструктур, концепція «потенціал», напівмарковських модель, імітаційна модель.

ОПРЕДЕЛЕНИЕ СИСТЕМНЫХ ХАРАКТЕРИСТИК ЭВОЛЮЦИИ КРИТИЧЕСКИХ ИНФРАСТРУКТУР НА ОСНОВЕ ТЕОРИИ ПОЛУМАРКОВСКИХ ПРОЦЕССОВ

А.В. Скатков, Н.А. Скаткова, В.С. Ловягин

Дается определение процесса эволюции критической инфраструктуры. В статье предоставляется, разработанная на основе теории полумарковских процессов, аналитическая модель эволюции критических инфраструктур. Приводится формула для расчета базовых системных характеристик эволюции (математического ожидания времени, необходимого для эволюции. Перечисленны некоторые недостатки аналитического подхода к решению проблемы исследования эволюции критически инфраструктур. Предложена имитационная модель эволюции как альтернативное решение проблемы.

Ключевые слова: эволюция критических инфраструктур, концепция «потенциал», полумарковская модель, имитационная модель.

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