UDC 621.3

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THE APPLICATION OF LINEAR PROGRAMMING METHODS FOR COMPUTER SYSTEMS DESIGN

The necessity of multicriterion problem of linear programming decision is considered in the given work. The limitation method of multicriterion problems solution is proposed. The example of multicriterion problem solution is shown to demonstrate the advantages of the limitation method.

linear programming, multicriterion problem, criterion function, simplex-method, automated design systems, relative wastage function, optimal decision, limitation method

Introduction

A wide range of questions is to be considered during the process of design of reliable and fault-tolerant systems and devices [1 – 4]. Nowadays the majority of computer system design problems are referred to multicriterion problems. The great number of facts is to be taken into account during the process of their solution. Moreover, the competition has become strained, so the project decisions are to be reached in the real-time operation mode. Decision taking in the computer system (CS) design process lies in the generation of possible structural and functional schemes of the designed computing device, in the analysis of these schemes and in the selection of the best one among them. The "correct" decision is such a variant of device scheme, which optimizes some criterion function (productivity, cost, etc.) in the environment of considered variants. With the appearance and the development of automation design systems of CS the mentioned processes of generation of different architecture versions and the processes of best decision choice are realized in the context of complex CS of electronic component design. But the generation is fulfilled not by the design system itself, but by the developers manually. In the connection with this fact, the problem of formalization and algorithmization of various versions of the design decision generation process appears to be urgent.

Publication analysis. The effectiveness of the automated design systems (ADS) can be essentially raised by the inclusion of optimization methods, allowing receiving the best of all possible decisions in the given concrete situation [1]. The problem of optimal object structure choice for the realization of the required functions is always urgent [2, 3]. It's known, that structural synthesis of the designed objects is a very laborious process, especially during the optimal decision research. To receive the optimal variant of the designed object structure the presence of its mathematical model is necessary. The mathematical model of the project represents the formal description of the set of object structures on the accepted detailed elaboration level. In that case the problem of structural synthesis reduces to the compromise variant choice on the computing set. The considerable time delays are required to solve the emergent logic combinatorial problems. In connection with this the problem of combinatorial provision of ADS arises. The combinatorial provision is natural to be created on the basis of sufficiently mutual mathematical models, which allow describing large problem complexes in a universal and compact form and constructing program packets to decide them by bearing on the unified arsenal of approaches and methods. The considerable part of formalized problems of structural synthesis of technical objects can be reduced to extreme criterion function value determination.

Problem statement. In lots of practical computer CS design problems, described by linear programming models, the decision choice, which was made only by one quality measure, can be unequal to the essence of the solving problem, as a lot of criteria have to be taken into account at the same time. For example, while CS designing, such factors as reliability, universality, cost, processing speed, constructive size, time for design are to be considered. Some of these factors are to be reduces to minimum in the design process, and some of them are be brought to maximum. Such a statement of the problem leads to the necessity of resolution of multicriterion linear programming problems.

The decision of linear programming multicreterion problems

Some set of criterion functions f_i(X), where

$$f_i(x) = c_i^T x = \sum_{j-1}^{n} c_{ij} x_j, i = \overline{1, M} \tag{1}$$

is given.

The first m functions have to be maximized, and the other (*M-m*) functions have to be minimized. Linear limitations

$$AX \le B^{k+1}, \tag{2}$$

$$x_j \ge 0, j = \overline{1, n} \tag{3}$$

are imposed on control action vector $X = \{x_j\}, j = \overline{1, n}$.

We accept the limitation method to solve this problem. The transformations, which lead the criteria to the nondimensional state, will assume the following expression:

a) for maximized criterion functions

$$W_i(f_i(x)) = \frac{C_i^T X_i^0 - C_i^T X}{C_i^T X_i^0 - C_i^T X_{f\min}}, \forall i \in I_1 = \{1, \ldots, m\}; \tag{4}$$

б) for minimized criterion functions

$$W_i(f_i(x)) = \frac{C_i^T X - C_i^T X_i^0}{C_i^T X_{f\max} - C_i^T X_i^0}, \quad \forall i \in I_2 = \{m+1, m+2, \ldots, M\}, \tag{5}$$

where X_i^0 is the solution which satisfies conditions (2), (3) and optimizes the *i*-criterion function.

$X_{f\min}(X_{f\max})$ is the solution, which minimizes (maximizes) the corresponding criterion function on the allowable set of decisions.

The compromise solution of the considered multicriterion problem is such effective solution *X*, which is characterized by equal and minimal weighted relational costs, that is

$$p_1 W_1(X) = p_2 W_2(X) = \ldots = p_m W_m(X) = k_{0\min}. \tag{6}$$

According to the limitation method, the required compromise solution can be found from linear inequation system solution for the minimum value of *k*-variable, for which this system is compatible:

$$\begin{aligned} &C_i^T X \ge C_i^T X_i^0 - \frac{k_0}{p_i}(C_i^T X_i^0 - C_i^T X_{i_{opt}}), \forall i \in I_1; \\ &C_i^T X \le C_i^T X_i^0 + \frac{k_0}{p_i}(C_i^T X_{iopt}^0 - C_i^T X_i), \forall i \in I_2; \\ &AX \le B \\ &x_j \ge 0, j = \overline{1, n}. \end{aligned} \tag{7}$$

The solution of linear inequation system is equivalent to the decision of the following problem of linear programming:

An expression

$$k_0 = x_{n+1} \tag{8}$$

has to be minimized, using limitations:

$$\begin{aligned} &\sum_{j=1}^{n} d_{1_j} x_j + d_{1,n+1} x_{n+1} + d_1 \ge 0; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\sum_{j=1}^{n} d_{ij} x_j + d_{i,n+1} x_{n+1} + d_i \ge 0; \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\sum_{j=1}^{n} d_{M_j} x_j + d_{M_{n+1}} x_{n+1} + d_M \ge 0; \end{aligned} \tag{9}$$

$$\left.\begin{array}{l}\sum\limits_{j=1}^{n} a_{ij}x_j - b_i \le 0\\[2mm] \sum\limits_{j=1}^{n} a_{kj}x_j - b_k \le 0\end{array}\right\} \qquad (10)$$

$$x_j \ge 0,\ j=\overline{1,n}\,, \qquad (11)$$

where

$$d_y = \begin{cases} p_i c_y, \forall j=\overline{1,n}; i\in I_1;\\ -\,p_i c_y,\ j=\overline{1,n}; i\in I_2; \end{cases} \qquad (12)$$

$$d_{1,n+1} = \begin{cases} \sum\limits_{j=1}^{n} c_y\,(x_y^{(0)} - x_{y_{\min}}), i\in I_1;\\ \sum\limits_{j=1}^{n} c_y\,(x_{y\max} - x_y^{(0)}), i\in I_2; \end{cases} \qquad (13)$$

$$d_{1,n+1} = \begin{cases} \sum\limits_{j=1}^{n} c_y\,(x_y^{(0)} - x_{y_{\min}}), i\in I_1;\\ \sum\limits_{j=1}^{n} c_y\,(x_{y\max} - x_y^{(0)}), i\in I_2. \end{cases} \qquad (14)$$

The example of multicriterion LP problem decision finding

Let's consider the example. The expression $f_1(x) = x_1 + 4x_2$ is to be maximized, the expression $f_2(x) = 3x_1 - x_2$ is to be minimized using limitations:

$$\begin{array}{l}\delta_1: x_1 + 2x_2 \ge 4;\\ \delta_2: 3x_1 + x_2 \ge 7;\\ \delta_3: -3x_1 + 5x_2 \le 17;\\ \delta_4: 5x_1 - x_2 \le 23;\\ \delta_5: 3x_1 - 4x_2 \le 7;\\ x_1, x_2 \ge 0.\end{array}$$

The permissible set of problem solutions $R(x)$ is represented on picture in the form of polygon ABCDE (fig. 1).

The optimal decision of this problem is easy to obtain graphically by $f_1(x)$ criterion. It is in the point C (x_1 = 6; x_2 = 7). Function value which corresponds to this solution is $f_1 = f(x_1 = 6; x_2 = 7) = 34$.

The minimal value is in the point E ($x_1 = 3$; $x_2 = 0{,}5$), $f_{1min} = 5$.

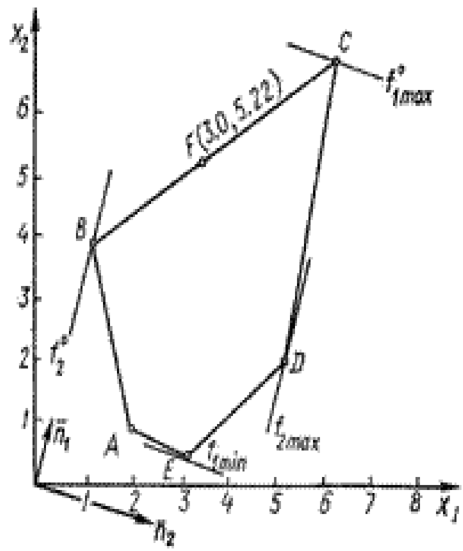

Fig. 1. Graphical decision of the multicriterion LP problem

According to the second criterion $f_2(x)$ the optimal decision will be in the point B (1, 4).

Function value $f_2^0 = f_2(1{,}4) = -1$ corresponds to it and the worst criterion $f_2(x)$ value is in the point D (5, 2): $f_2 = 13$. The section BC represents an effective plan.

Let's consider the case of equivalent criteria. This means that $p_1 = p_2$ and the compromise solution which provides minimal equal relative wastage will be looked for.

Relative wastage functions are:

$$W_1' = p_1 W_1(x) = \frac{1}{2}\frac{f_1^0 - f_1(x)}{f_1^0 - f_{1opt}} = \frac{1}{2}\frac{34 - x_1 - 4x_2}{29},$$

$$W_2' = p_2 W_2(x) = \frac{1}{2}\frac{f_2(x) - f_2^0}{f_{2opt} - f_2^0} = \frac{1}{2}\frac{3x_1 - x_2 + 1}{14}.$$

Let's note the equivalent linear programming problem according to (7) – (10):

To minimize $x_3 = k_0$ on conditions that

$$p_1\frac{f_1^0 - f_1(x)}{f_1^0 - f_{1opt}} = \frac{1}{2}\frac{34 - x_1 - 4x_2}{29} \le x_3;$$

$$p_2\frac{f_2(x) - f_2^0}{f_{2opt} - f_2^0} = \frac{1}{2}\frac{3x_1 - x_2 + 1}{14} \le x_3;$$

$$\left.\begin{array}{l} x_1 + 2x_2 \ge 4 \\ 3x_1 + x_2 \ge 7 \\ -3x_1 + 5x_2 \le 17 \\ 5x_1 - x_2 \le 23 \\ 3x_1 - 4x_2 \le 7 \end{array}\right\}$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

After elementary transformations the problem is brought to the following state.

To minimize x_3 on the assumption of

$$\left.\begin{array}{l} x_1 + 4x_2 + 58x_3 \ge 34 \\ -3x_1 + x_2 + 28x_3 \ge 1 \\ x_1 + 2x_2 \ge 4 \\ 3x_1 + x_2 \ge 7 \\ 3x_1 - 5x_2 \ge -17 \\ -5x_1 + x_2 \ge -23 \\ -3x_1 + 4x_2 \ge -7 \end{array}\right\}$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

The given problem has $n = 3$ variables and $m = 7$ limitations. As $m > n$, use the transition to dual problem conjugated with (7) – (9) for the purpose of simplification of optimal solution research. It has such appearance.

To maximize $(34y_1 + y_2 + 4y_1 + 7y_4 - 17y_5 - 23y_6 - 7y_7)$ on conditions that

$$\begin{gathered} y_1 - 3y_2 + y_3 + 3y_4 + 3y_5 - 5y_6 - 3y_7 \le 0; \\ 4y_1 + 1y_2 + 2y_3 + y_4 - 5y_5 + 1y_6 + 4y_7 \le 0; \\ 58y_1 + 28y_2 \le 1; \\ y_1 \ge 0, y_2 \ge 0, y_3 \ge 0. \end{gathered}$$

This problem may be solved by simplex-method. After the application of simplex-method to the achievement of the decision of the given problem we get the optimal decision of the direct multicriterion problem. We have:

$$x_1^0 = \left|\Delta_{7-M}^{(dw)}\right| = \left|\Delta_8^{(dw)}\right| = 3{,}03;$$
$$x_2^0 = \left|\Delta_9^{(dw)}\right| = 5{,}22;$$
$$x_3^0 = \left|\Delta_{10}^{(dw)}\right| = k_0 = 0{,}175.$$

The point F in the BC section (fig. 1) corresponds to this decision. The minimal relative wastages and minimal deviation of optimal value on both of criterions conform to this solution:

$$W_1\left(f_1\left(x_1^0, x_2^0\right)\right) = \frac{1}{2}\,\frac{f_1^0 - f_1\left(x_1^0, x_2^0\right)}{f_1^0 - f_{1opt}} = 0{,}175 = k_0;$$
$$W_2\left(f_2\left(x_1^0, x_2^0\right)\right) = \frac{1}{2}\,\frac{f_2\left(x_1^0, x_2^0\right) - f_2^0}{f_{2opt} - f_2^0} = 0{,}175 = k_0.$$

Conclusion

The obtained results are the most appropriate to be used during the process of computer technique tools design on the basis of programmable logic arrays [5], as the sharp increase of integrated circuit volumes requires formal methods of project decision analysis. The usage of the developed methods allows increasing reliability and fault-tolerance degree of the devices, which are being created on the basis of CPLD.

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Поступила в редакцию 21.02.2006

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