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Application of the Generalized Taylor – Birkhoff Series for Solving of the Initial Value Problem for Ordinary Differential Equations

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In this paper we propose application of the modified generalized Taylor – Birkhoff series, based on the atomic function *up(x)*, for solving of the initial value problem for the ordinary differential equations and systems of differential equations. The explicit formulas for the basic functions of the modified generalized Taylor – Birkhoff series up to the third order are given.

Keywords: antiderivative, initial value problem for ODE, generalized Taylor – Birkhoff series, basic functions of the atomic generalized Taylor – Birkhoff series.

1. Statement of the problem and analysis of recent research and publications

Consider the initial value problem for the differential equation of the first order

$$y'(x) = F(x, y(x)),\qquad y(0) = y_0. \tag{1}$$

It equivalent to the Volterra integral equation

$$y(x) = y_0 + \int_0^x F(t, y(t))dt\,. \tag{2}$$

If we solve this equation by iteration method, assuming

$$y_0(x) = y_0$$

and

$$y_{n+1}(x) = y_0 + \int_0^x F(t, y_n(t))dt \tag{3}$$

then for every iteration step we should find the antiderivative of a function $F(x, y_n(x))$.

We cannot use usual quadrature formulas in this case, since the upper limit of the integral is variable. Classical Taylor series [1–4] has some restrictions in application, firstly since its radius of convergence may be insufficient, and mainly because the substitution of the power series instead $y_n(t)$ into a function $F(t, y_n(t))$ requires further transformations for the obtaining the power series under the integral sign.

In [5–10] so called generalized Taylor – Birkhoff series for the expanding of infinitely differentiable functions of some Roumieu spaces was introduced:

$$F(x) = \sum_{n=0}^{\infty} \sum_{k \in N_n} F^{(n)}(x_{n,k})\varphi_{n,k}(x)\,,$$

where functions $\varphi_{n,k}(x)$ are the basic functions of generalized Taylor – Birkhoff series, which can be expressed as linear combinations of translates of atomic function [11–13]

$$up(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}\prod_{k=1}^{\infty}\frac{\sin t2^{-k}}{t2^{-k}}dt,$$

which is a solution with a compact support of the functional–differential equation

$$y'(x) = 2y(2x+1) - 2y(2x-1).$$

The points $x_{n,k}$ are defined as follows:

$$\text{for } n = 0 \;\; x_{0,k} = k,$$

$$\text{for } n > 0 \; x_{n,k} = k2^{n-1}.$$

So $x_{1,k} = k$, $x_{2,k} = \frac{k}{2}$, $x_{3,k} = \frac{k}{4}$ and so on.

With the help of modified generalized Taylor – Birkhoff series method of finding antiderivatives was proposed in [14]. The corresponding modification of the series is made to avoid using values of F(x) at $x \neq 0$ and the necessity to calculate the definite integrals.

In this paper we propose to use this modified series for solving of the initial value problem for the ordinary differential equations of the first order

$$y'(x) = F(x, y(x)), \quad y(x_0) = y_0$$

and systems of ordinary differential equation.

2. Modification of the generalized Taylor – Birkhoff series and the formulas for the basic functions of the modified series

In this process the basic functions of the generalized Taylor – Birkhoff series $\varphi_{n,k}(x)$ are substituted by the modified basic functions $\tilde{\varphi}_{n,k}(x)$. Namely, instead of defining the values of a function $F(x)$, represented by the generalized Taylor – Birkhoff series, at the points $k \neq 0$, we define the derivative of this function at the points $k - 1/2,\, k > 0,\, k + 1/2, k < 0$. Thus the modified $\tilde{x}_{1,k} = \frac{k}{2}$.

The corresponding modified basic functions for $k > 0$ are of the form

$$\tilde{\varphi}_{1,k-1/2}(x) = \begin{cases} 0.5up(x-k), x \le k, \\ 0.5, x > k. \end{cases}$$

Similarly we build the basic functions $\tilde{\varphi}_{1,k+1/2}(x)$ for $k < 0$.

Further, all the other modified basic functions $\tilde{\varphi}_{n,k}(x)$ we obtain from the standard basic functions by subtraction of functions $\alpha \cdot \varphi_{1,k-1/2}(x)$, $\beta \cdot \varphi_{1,k+1/2}(x)$ where α, β we choose to make the first derivatives of $\tilde{\varphi}_{n,k}(x)$ at the points $k - 1/2,\, k > 0,\, k + 1/2\;, k < 0$ equal to 0.

Give the formulas for the modified basic functions $\tilde{\varphi}_{n,k}(x)$, which are necessary for the expansion in the modified generalized Taylor – Birkhoff series up to the third order at [0,1]:

$$\tilde{\varphi}_{0,0}(x) = up(x) + up(x-1), \quad x \in [0,1],$$

$$\tilde{\varphi}_{1,1/2}(x) = 0.5up(x-1), \quad x \in [0,1],$$

$$\tilde{\varphi}_{1,1}(x) = 0.5up(x - \frac{3}{2}), \quad x \in [0,1],$$

$$\tilde{\varphi}_{1,0}(x)=\frac{1}{4}up(x)-\frac{1}{2}up(x+\frac{1}{2})+\frac{1}{4}up(x-1),\quad x\in[0,1],$$

$$\tilde{\varphi}_{2,0}(x)=\frac{13}{576}up(x)+\frac{13}{576}up(x-1)-\frac{1}{16}up(x+\frac{1}{2})+\frac{1}{8}up(x+\frac{3}{4}),\quad x\in[0,1],$$

$$\tilde{\varphi}_{2,1}(x)=-\frac{1}{16}up(x-\frac{3}{2})+\frac{1}{8}up(x-\frac{7}{4}),\quad x\in[0,1],$$

$$\tilde{\varphi}_{2,\frac{1}{2}}(x)=\begin{cases}\frac{13}{72\cdot 8}up(x+\frac{1}{2})-\frac{13}{72\cdot 8}up(x)+\frac{13}{72\cdot 8}up(x-\frac{1}{2})-\\ \quad -\frac{49}{72\cdot 8}up(x-1)+\frac{1}{8}up(x-\frac{5}{4}), x\in\left[0,\frac{1}{2}\right],\\ \frac{1}{8}up(x+\frac{1}{4})-\frac{49}{72\cdot 8}up(x)+\frac{13}{72\cdot 8}up(x-\frac{1}{2})-\\ \quad -\frac{13}{72\cdot 8}up(x-1)+\frac{13}{72\cdot 8}up(x-\frac{3}{2}), x\in\left[\frac{1}{2},1\right],\end{cases}$$

$$\tilde{\varphi}_{3,0}(x)=\frac{17}{288\cdot 64}up(x-1)+\frac{17}{288\cdot 64}up(x)-\frac{13}{72\cdot 64}up(x+\frac{1}{2})+$$
$$+\frac{1}{2\cdot 64}up(x+\frac{3}{4})-\frac{1}{64}up(x+\frac{7}{4}),\quad x\in[0,1],$$

$$\tilde{\varphi}_{3,\frac{3}{4}}(x)=\begin{cases}-\frac{17}{288\cdot 64}up(x-\frac{5}{4})+\frac{911}{64\cdot 6912}up(x-1)-\frac{17}{64\cdot 288}up(x-\frac{3}{4})-\\ -\frac{299}{64\cdot 6912}up(x-\frac{1}{2})+\frac{503}{64\cdot 6912}up(x)-\frac{707}{6912\cdot 64}up(x+\frac{1}{2})+\\ +\frac{17}{288\cdot 64}up(x+\frac{3}{4}), x\in[0,\frac{1}{2}],\\ \frac{1}{64}up(x-\frac{13}{8})-\frac{5003}{64\cdot 6912}up(x-\frac{3}{2})+\frac{13}{72\cdot 64}up(x-\frac{5}{4})+\\ +\frac{83}{64\cdot 6912}up(x-1)-\frac{17}{288\cdot 64}up(x-\frac{3}{4})-\frac{299}{64\cdot 6912}up(x-\frac{1}{2})+\\ +\frac{1331}{64\cdot 6912}up(x)-\frac{23}{96\cdot 64}up(x+\frac{1}{4}), x\in[\frac{1}{2};\frac{3}{4}],\\ \frac{23}{96\cdot 64}up(x-\frac{7}{4})-\frac{1547}{64\cdot 6912}up(x-\frac{3}{2})+\frac{83}{64\cdot 6912}up(x-1)+\\ +\frac{17}{288\cdot 64}up(x-\frac{3}{4})-\frac{299}{6912\cdot 64}up(x-\frac{1}{2})-\frac{13}{72\cdot 64}up(x-\frac{1}{4})+\\ +\frac{4787}{64\cdot 6912}up(x)-\frac{1}{64}up(x+\frac{1}{8}), x\in[\frac{3}{4};1],\end{cases}$$

$$\tilde{\varphi}_{3,\frac{1}{2}}(x)=\begin{cases}-\frac{17}{288\cdot 64}up(x+\frac{1}{2})+\frac{17}{288\cdot 64}up(x)-\frac{17}{288\cdot 64}up(x-\frac{1}{2})+\\+\frac{69}{288\cdot 64}up(x-1)-\frac{1}{2\cdot 64}up(x-\frac{5}{4})+\\+\frac{1}{64}up(x-\frac{11}{8}),\quad x\in\left[0,\frac{1}{2}\right],\\-\frac{1}{64}up(x+\frac{3}{8})+\frac{1}{2\cdot 64}up(x+\frac{1}{4})-\frac{35}{288\cdot 64}up(x)+\\+\frac{17}{288\cdot 64}up(x-\frac{1}{2})+\frac{17}{288\cdot 64}up(x-1)+\\+\frac{17}{288\cdot 64}up(x-\frac{3}{2}),\quad x\in\left[\frac{1}{2},1\right],\end{cases}$$

$$\tilde{\varphi}_{3,1}(x)=\frac{13}{72\cdot 64}up(x-\frac{3}{2})-\frac{1}{2\cdot 64}up(x-\frac{7}{4})+\frac{1}{64}up(x-\frac{15}{8}),\quad x\in[0,1],$$

$$\tilde{\varphi}_{3,\frac{1}{4}}(x)=\begin{cases}\frac{1}{64}up(x-\frac{9}{8})-\frac{5435}{64\cdot 6912}up(x-1)+\frac{13}{72\cdot 64}up(x-\frac{3}{4})+\\+\frac{299}{64\cdot 6912}up(x-\frac{1}{2})-\frac{17}{64\cdot 288}up(x-\frac{1}{4})-\frac{515}{64\cdot 6912}up(x)+\\+\frac{1547}{64\cdot 6912}up(x+\frac{1}{2})-\frac{23}{96\cdot 64}up(x+\frac{3}{4}),x\in\left[0,\frac{1}{4}\right],\\\frac{29}{64\cdot 96}up(x-\frac{5}{4})-\frac{1979}{64\cdot 6912}up(x-1)+\frac{299}{64\cdot 6912}up(x-\frac{1}{2})+\\+\frac{17}{288\cdot 64}up(x-\frac{1}{4})-\frac{515}{64\cdot 6912}up(x)-\frac{13}{72\cdot 64}up(x+\frac{1}{4})+\\+\frac{5003}{6912\cdot 64}up(x+\frac{1}{2})-\frac{1}{64}up(x+\frac{5}{8})-\frac{23}{96\cdot 64}up(x+\frac{3}{4}),x\in\left[\frac{1}{4},\frac{1}{2}\right],\\-\frac{17}{288\cdot 64}up(x-\frac{7}{4})+\frac{707}{6912\cdot 64}up(x-\frac{3}{2})-\frac{1151}{64\cdot 6912}up(x-1)+\\+\frac{299}{64\cdot 6912}up(x-\frac{1}{2})+\frac{17}{288\cdot 64}up(x-\frac{1}{4})-\frac{1343}{64\cdot 6912}up(x)+\\+\frac{17}{288\cdot 64}up(x+\frac{1}{4})+\frac{5003}{6912\cdot 64}up(x+\frac{1}{2})-\frac{1}{64}up(x+\frac{5}{8})-\\-\frac{23}{96\cdot 64}up(x+\frac{3}{4}),x\in\left[\frac{1}{2},1\right],\end{cases}$$

3. Iteration method

Consider iteration method for the Volterra integral equation (2) which is equivalent to the initial value problem (1):

$$y_{n+1}(x) = y_0 + \int_0^x F(t, y_n(t))dt\,.$$

Expand the antiderivative

$$\Phi_{n+1}(x) = \int_0^x F(t, y_n(t))dt$$

in the modified generalized Taylor – Birkhoff series up to the third order at [0,1]. Then since $\Phi_{n+1}(0) = 0$ we obtain

$$y_{n+1}(x) = y_0 + \sum_{k=0}^{2} [\Phi'_{n+1}(\frac{k}{2})\tilde{\varphi}_{1,\frac{k}{2}}(x) + \Phi''_{n+1}(\frac{k}{2})\tilde{\varphi}_{2,\frac{k}{2}}(x)] + \\ + \sum_{k=0}^{4} \Phi'''_{n+1}(\frac{k}{4})\tilde{\varphi}_{3,\frac{k}{4}}(x)\,.$$

Given that

$\Phi'_{n+1}(x) = F(x, y_n(x))$,

$\Phi''_{n+1}(x) = F'_x(x, y_n(x)) + F'_y(x, y_n(x))y'_n(x)$,

$$\Phi_{n+1}(x) = F''_{xx}(x, y_n(x)) + 2F''_{xy}(x, y_n(x))y'_n(x) + \\ + F''_{yy}(x, y_n(x))(y'_n(x))^2 + F'_y(x, y_n(x))y''_n(x)$$

and denoting

$A_{1,k} = F(\frac{k}{2}, y_0), \quad k = 0,1,2$;

$A_{n+1,k} = F(\frac{k}{2}, y_n(\frac{k}{2})), \quad k = 0,1,2, \; n > 0$;

$B_{1,k} = F'_x(\frac{k}{2}, y_0), \quad k = 0,1,2$

$B_{n+1,k} = F'_x(\frac{k}{2}, y_n(\frac{k}{2})) + F'_y(\frac{k}{2}, y_n(\frac{k}{2}))y'_n(\frac{k}{2}), \quad k = 0,1,2, \; n > 0$;

$C_{1,k} = F''_{xx}(\frac{k}{4}, y_0), \quad k = 0,1,2,3,4$

$$C_{n+1,k} = F''_{xx}(\frac{k}{4}, y_n(\frac{k}{4})) + 2F''_{xy}(\frac{k}{4}, y_n(\frac{k}{4}))y'_n(\frac{k}{4}) + \\ + F''_{yy}(\frac{k}{4}, y_n(\frac{k}{4}))(y'_n(\frac{k}{4}))^2 + F'_y(\frac{k}{4}, y_n(\frac{k}{4}))y''_n(\frac{k}{4}),$$

$k = 0,1,2,3,4, \quad n > 0,$

we obtain

$$y_{n+1}(x) = y_0 + \sum_{k=0}^{2}[A_{n+1,k}\tilde{\varphi}_{1,\frac{k}{2}}(x) + B_{n+1,k}\tilde{\varphi}_{2,\frac{k}{2}}(x)] + \sum_{k=0}^{4} C_{n+1,k}\tilde{\varphi}_{3,\frac{k}{4}}(x),$$

where recurrent formulas for the coefficients $A_{n+1,k}$, $B_{n+1,k}$, $C_{n+1,k}$ are

$$A_{n+1,k} = F(\frac{k}{2}, y_0 + \sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{2}) + B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{2})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{2})), \quad k = 0,1,2, \quad n > 0,$$

$$B_{n+1,k} = F'_x(\frac{k}{2}, y_0 + \sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{2}) + B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{2})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{2})) +$$

$$+ F'_y(\frac{k}{2}, y_0 + \sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{2}) + B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{2})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{2})) \cdot$$

$$\cdot (\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}'_{1,\frac{s}{2}}(\frac{k}{2}) + B_{n,s}\tilde{\varphi}'_{2,\frac{s}{2}}(\frac{k}{2})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}'_{3,\frac{s}{4}}(\frac{k}{2})),$$

$$k = 0,1,2, \; n > 0,$$

$$C_{n+1,k} = F''_{xx}(\frac{k}{4}, y_0 + \sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{4}) + B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{4})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{4})) +$$

$$+ 2F''_{xy}(\frac{k}{4}, y_0 + \sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{4}) + B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{4})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{4})) \cdot$$

$$\cdot (\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}'_{1,\frac{s}{2}}(\frac{k}{4}) + B_{n,s}\tilde{\varphi}'_{2,\frac{s}{2}}(\frac{k}{4})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}'_{3,\frac{s}{4}}(\frac{k}{4})) +$$

$$+ F''_{yy}(\frac{k}{4}, y_0 + \sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{4}) + B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{4})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{4})) \cdot$$

$$\cdot (\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}'_{1,\frac{s}{2}}(\frac{k}{4}) + B_{n,s}\tilde{\varphi}'_{2,\frac{s}{2}}(\frac{k}{4})] + \sum_{s=0}^{4} C_{n,s}\tilde{\varphi}'_{3,\frac{s}{4}}(\frac{k}{4}))^2 +$$

$$+F'_y(\frac{k}{4}, y_0+\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{4})+B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{4})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{4}))\cdot$$

$$\cdot(\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}''_{1,\frac{s}{2}}(\frac{k}{4})+B_{n,s}\tilde{\varphi}''_{2,\frac{s}{2}}(\frac{k}{4})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}''_{3,\frac{s}{4}}(\frac{k}{4})).$$

Consider, for example, the initial value problem for the Riccati equation:

$$y(x)=\pm y^2+f(x),$$
$$y(0)=y_0.$$

Then

$$F(x,y)=\pm y^2+f(x),$$
$$F'_x=f'(x),$$
$$F''_{xx}=f''(x),$$
$$F''_{xy}=0,$$
$$F'_y=\pm 2y,$$
$$F''_{yy}=\pm 2.$$

And we obtain the following recurrent formulas for the coefficients $A_{n+1,k}$, $B_{n+1,k}$, $C_{n+1,k}$:

$$A_{n+1,k}=f(\frac{k}{2})\pm[y_0+\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{2})+B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{2})]+$$

$$+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{2}))]^2,\quad k=0,1,2,\quad n>0,$$

$$B_{n+1,k}=f'(\frac{k}{2})\pm 2[y_0+\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{2})+B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{2})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{2})]\cdot$$

$$\cdot(\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}'_{1,\frac{s}{2}}(\frac{k}{2})+B_{n,s}\tilde{\varphi}'_{2,\frac{s}{2}}(\frac{k}{2})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}'_{3,\frac{s}{4}}(\frac{k}{2})),$$

$$k=0,1,2,\ n>0,$$

$$C_{n+1,k}=f''(\frac{k}{4})\pm 2(\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}'_{1,\frac{s}{2}}(\frac{k}{4})+B_{n,s}\tilde{\varphi}'_{2,\frac{s}{2}}(\frac{k}{4})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}'_{3,\frac{s}{4}}(\frac{k}{4}))^2\pm$$

$$\pm 2(y_0+\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}_{1,\frac{s}{2}}(\frac{k}{4})+B_{n,s}\tilde{\varphi}_{2,\frac{s}{2}}(\frac{k}{4})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}_{3,\frac{s}{4}}(\frac{k}{4}))\cdot$$

$$\cdot\left(\sum_{s=0}^{2}[A_{n,s}\tilde{\varphi}''_{1,\frac{s}{2}}(\frac{k}{4})+B_{n,s}\tilde{\varphi}''_{2,\frac{s}{2}}(\frac{k}{4})]+\sum_{s=0}^{4}C_{n,s}\tilde{\varphi}''_{3,\frac{s}{4}}(\frac{k}{4})\right).$$

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Застосування узагальненого ряду Тейлора – Біркгофа для розв’язування задачі Коші для звичайних диференціальних рівнянь

У роботі запропоновано застосування модифікованого узагальненого ряду Тейлора – Біркгофа, побудованого на основі атомарної функції $up(x)$, для розв’язування задачі Коші для звичайних диференціальних рівнянь та систем диференціальних рівнянь. Наведено явні формули для базисних функцій модифікованого узагальненого ряду Тейлора – Біркгофа до третього порядку включно.

Ключові слова: первісна, задача Коші для звичайних диференціальних рівнянь, узагальнений ряд Тейлора – Біркгофа, базисні функції узагальненого ряду Тейлора – Біркгофа.

Применение обобщенного ряда Тейлора – Биркгофа для решения задачи Коши для обыкновенных дифференциальных уравнений

В работе предложено применение модифицированного обобщенного ряда Тейлора – Биркгофа, построенного на основе атомарной функции $up(x)$, для решения задачи Коши для обыкновенных дифференциальных уравнений и систем дифференциальных уравнений. Приведены явные формулы для базисных функций модифицированного обобщенного ряда Тейлора – Биркгофа до третьего порядка включительно.

Ключевые слова: первообразная, задача Коши для обыкновенных дифференциальных уравнений, обобщенный ряд Тейлора – Биркгофа, базисные функции обобщенного ряда Тейлора – Биркгофа.

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